Double-Layer Gypsum Panels: Prediction of the Sound Reduction Index Using the Transfer Matrix Method

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Abstract

Gypsum board walls are widely used in buildings today. A possible way to considerably increase the sound insulation performances of such light-weight walls is to apply a double or triple layer of screwed boards separated from each other. The separation between the boards prevents the decrease of the critical frequency towards lower values, while retaining the improvement of the sound insulation performances provided by the double layer. In this paper the loss of acoustic insulation performances due to the thin air layer between the coupled boards is studied and a modelling technique based on the transfer matrix method is used to simulate the acoustic behaviour of the resulting structure. The simulations are compared with laboratory measurements carried out according to the ISO 10140 series standards, and the transfer matrix approach is found to be suitable to describe the problem, provided that a modified model for the air gap between the boards is used.

1. Introduction

The early application of gypsum-based walls as construction materials dates back to the first half of the twentieth century in the US, and since then it has been continuously developed due to the unique characteristics of this material: low cost, good workability, relatively low weight and quick laying. The issuance of strict regulations regarding thermal and acoustic quality has further increased the preference for these materials in different sectors, especially the building construction, as they are particularly suitable to the tailored optimisation that new technologies require nowadays. A typical optimisation problem in building construction demands that both thermal and acoustic requirements are met: a complex task (Di Bella et al., 2015) in which an improvement in either aspect does not necessarily imply the improvement in the other (Bettarello et al., 2010; Caniato et al., 2015; Caniato et al., 2019; Caniato et al., 2020; Di Bella et al., 2014). Therefore, the availability of simple tools to quickly and reliably predict the acoustic behaviour of the partition is of a certain importance.

Many authors have studied the sound insulation behaviour of lightweight walls made of gypsum board, gypsum fibreboards and wood panels featuring an air gap filled with mineral or glass wool. Abd El Gawad Saif and Seddek (2010) experimentally evaluated the sound insulation performances of different gypsum board-based partitions and investigated the influence of several factors on the resulting sound transmission class. Uris et al. (2002) studied the influence of the number and spacing of point connections due to screws on the sound reduction index of lightweight partitions. It was found that the screw spacing is more important in double-leaf walls than in uncoupled double walls due to structural transmission through the frame. Kim et al. (2010) identified several factors that influence the sound insulation performance of gypsum-board walls, such as the method used in attaching gypsum boards, the addition of a further board, the installation of sound absorbing material, the curing time of the plaster and the type of studs used. Roozen et al. (2015) found indeed that even the tightening level of the studs can considerably affect the insulation properties.

In terms of mathematical models, one of the best-known analytical approaches derives from Cremer's classic theory (1942), and is applicable to homogeneous plates. The introduction of a frequency-dependent bending stiffness by Nilsson (1990) also allows the approach to be extended to complex structures, such as sandwich panels, whose

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bending stiffness can be measured experimentally and fed as input to the model (Nilsson and Nilsson, 2002). This method is used by Piana et al. (2014), and Piana et al. (2017) to determine the transmission loss of dry-wall panels displaying orthotropic and sandwich-like features, showing good agreement with standardised measurement results. Another possible approach is provided by the transfer matrix method (TMM), which is particularly suitable to model multi-layer structures. The modelling consists in two steps: representing each layer with a transfer matrix, whose structure depends on the material, and composing the matrices to form the overall transfer matrix that relates the input and output acoustic states (Vigran, 2008).

When two identical gypsum walls are screwed (but not glued) together, and separated by a thin air gap, in the case of a diffuse sound field, the resulting structure exhibits a doubled mass per unit area but the critical frequency is still that of a single-layer structure. This aspect leads to an increase in the performance of the double-layer solution with respect to a single-layer solution of equal mass. On the other hand, some issues in the modelling of the sound insulation behaviour arise if a narrow air gap is left between the two gypsum panels as is the practice. This paper examines the sound insulation performances of double-layered gypsum board walls with different thicknesses by investigating the influence of a thin layer of air between the two plates through the transfer matrix method. In the following section, the transfer matrix method is presented with reference to the case at hand. Section 3 describes the panels used for the evaluation in terms of geometry and measured sound reduction indices, which the model results are compared to in Section 4. Finally, the conclusions are drawn and possible future developments are identified.

2. Double-Layer Gypsum Board Model

2.1 Transfer Matrix Method

The transfer matrix method (TMM) for multilayer acoustical systems allows wave motion in a single direction (Vigran, 2008). For this reason, the structure must be "thin", in that its thickness must be small enough with respect to the wavelength for the wave motion inside the structure to be negligible. In this hypothesis, each i-th layer of the multilayer structure (see Fig. 1) can be represented by a 2-by-2 matrix describing the relationship between the inlet and outlet acoustical quantities – typically, pressure p and particle velocity u:

$$\begin{bmatrix} p_{i-1} \\ u_{i-1} \end{bmatrix} = \begin{bmatrix} T_{i,11} & T_{i,12} \\ T_{i,21} & T_{i,22} \end{bmatrix} \begin{bmatrix} p_i \\ u_i \end{bmatrix} = \boldsymbol{T}_i \begin{bmatrix} p_i \\ u_i \end{bmatrix} \quad (1)$$

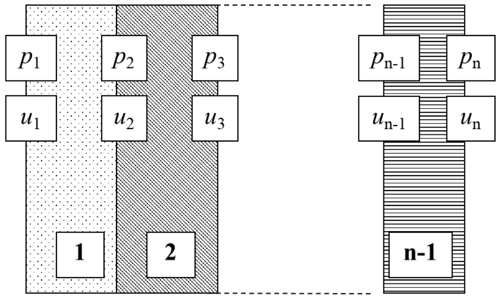

Fig. 1 – Multilayer structure

The elements of the matrix depend on the characteristics of the layer. For a fluid layer such as air, the fundamental quantities the transfer matrix depends on are the wavenumber in the fluid, k, or the propagation coefficient $\Gamma = \mathrm{j}k$, and the characteristic impedance Z_c. Assuming oblique incidence at angle θ with respect to the normal, the transfer matrix of a porous layer can be written as

$$\boldsymbol{T}_{\text{fluid}} = \\ = \begin{bmatrix} \cosh(\Gamma d \cdot \cos(\theta)) & \frac{Z_c \sinh(\Gamma d \cdot \cos(\theta))}{\cos(\theta)} \\ \frac{\sinh(\Gamma d \cdot \cos(\theta))}{Z_c}\cos(\theta) & \cosh(\Gamma d \cdot \cos(\theta)) \end{bmatrix} \quad (2)$$

where d is the thickness of the layer. A similar formulation also holds for porous layers, provided that the porosity is accounted for in the anti-diagonal terms.

The transfer matrix for a thin panel, such as a gypsum board, basically depends on the wall impedance of the element, Z_p, defined as the ratio between the sound pressure difference across the layer and the velocity. If the velocity is assumed to be equal on both sides of the panel, the transfer matrix can be written as

$$\boldsymbol{T}_{\text{panel}} = \begin{bmatrix} 1 & Z_p \\ 0 & 1 \end{bmatrix} \quad (3)$$

The matrices for the individual layers can subsequently be multiplied to obtain the matrix describing the full multilayer structure:

$$\boldsymbol{T} = \boldsymbol{T}_1 \cdot \boldsymbol{T}_2 \cdot \ldots \cdot \boldsymbol{T}_\mathrm{n} \tag{4}$$

The transmission coefficient τ can be obtained from the overall transfer matrix:

$$\tau = \frac{2\exp(jk_0 d_\mathrm{tot})}{T_{11}+\frac{T_{12}}{\rho_0 c_0}+\rho_0 c_0 T_{21}+T_{22}} \tag{5}$$

and the sound reduction index is $R = 10 \log_{10} (1/\tau)$.

2.2 Modelling the Layers

In modelling the air layer, one must consider that using Equation (2) is suitable for large cavities whose effect can be seen in the audible frequency range. This effect causes the presence of a resonance frequency due to the mass-spring-mass system. Above this resonance frequency, a typical 12 dB/octave slope can be observed. When two gypsum boards are connected together, the air gap is so thin, compressed and damped that the only effect is the introduction of a flow resistivity to the air trying to flow through the space between the vibrating boards. For this reason, instead of modelling the gap between the panels as an air layer, it was modelled as a small cavity filled with a porous material with low-air-flow-resistivity. If the frame of the porous layer can be considered as motionless, a simplified model for the porous layer can be applied. The porous layer is then modelled as an equivalent fluid. The losses in the air gap were taken into account by introducing a flow resistivity r and a complex propagation coefficient Γ_c and a complex characteristic impedance Z_c are calculated. A Delany-Bazley-like model for porous materials was used to calculate the two characterising parameters (Mechel, 2008):

$$Z_c = Z_0\left[1 + a\left(\frac{\rho_0 f}{r}\right)^{-b} - jc\left(\frac{\rho_0 f}{r}\right)^{-d}\right] \tag{6}$$

$$\Gamma_c = \frac{\omega}{c_0}\left[a'^{\left(\frac{\rho_0 f}{r}\right)^{-b'}} + j\left(1 + c'\left(\frac{\rho_0 f}{r}\right)^{-d'}\right)\right] \tag{7}$$

An air flow resistivity of 2000 (Pa s/m^2) and a thickness of 1 mm were considered. As a first approximation the following values were considered for the model parameters: *a=b=a'=b'=c'=d'=0* and *c=d=1*.
In terms of the gypsum board layers, only the wall impedance Z_p needs to be modelled. This can be done through Cremer's theory for homogeneous panels:

$$Z_\mathrm{p} = j\omega\mu\left[1 - (1 + j\eta)\left(\frac{f}{f_\mathrm{c}}\right)^2 \sin^4(\theta)\right] \tag{8}$$

Here, μ is the mass per unit area of the panel, η is the loss factor of the structure, and f_c is the critical frequency, which depends on the bending stiffness D through the expression

$$f_\mathrm{c} = \frac{c^2}{2\pi}\sqrt{\frac{\mu}{D}} \tag{9}$$

D being the bending stiffness of the panel. Here, the physical characteristics of the gypsum board were obtained through an inverse analysis starting from the measured mass per unit area of the panels. The modulus of elasticity and the loss factor can be derived by technical datasheet of the material and by dedicated measurements, where available. As an alternative, meaningful indications can also be obtained by measured sound reduction index curves.

3. Experimental Analysis

3.1 ISO 10140 Measurements

In order to validate the predictions provided by the model, gypsum board pairs with three different thicknesses were screwed together and some ad hoc measurements on the three structures were carried out in sound transmission suites following the procedure of ISO 10140 international standard series.
In particular, the standard requires that a diffuse sound fields is established in two adjacent rooms. Through measurements of the sound pressure level and of the sound absorption characteristics of the receiving room, the sound reduction index R can be determined as:

$$R = L_1 - L_1 + 10\log_{10}\left(\frac{S}{A}\right) \tag{10}$$

where A is the equivalent absorption area in the receiving room calculated as:

$$A = 0.16\frac{V}{t_r} \tag{11}$$

and t_r is the reverberation time of the receiving room.

3.2 Description of the Panels

This paper investigates the sound reduction index of gypsum board panels which have a thickness of 9.5 mm, 12.5 mm and 15 mm, mounted as a single or double layer and fastened together by using 20 screws (4 columns × 5 rows) on the panel surface (Table 1).

Table 1 – Gypsum board characteristics

Thickness (mm)	μ (kg/m²)	ρ (kg/m³)
9.5	6.9	730
12.5	9.0	721
15	12.5	833

The sound reduction index of the different panels was determined on 1700 mm × 1090 mm specimens (Fig. 2). In order to avoid sound leakage through the sides of the specimens fitted into the wall dividing the two rooms, a sealant (Perennator TX 2001 S) was used to close the air gap between the frame and the plate. The sound pressure levels in the emitting and receiving rooms were measured by using a L&D 824 sound level meter and subsequently space-averaged. The reverberation time of the receiving room was measured using the interruption of stationary pink noise technique.

Fig. 2 – Installation of the panel according to ISO 10140

The results of the theoretical model were compared with sound reduction index measurements performed in sound transmission rooms according to the ISO 10140 series standards.

Fig.s 3, 4 and 5 show the comparison between the results of the simulations and the predictions carried out by using the transfer matrix method for the 9.5 mm, the 12.5 mm and the 15 mm double-panel gypsum boards:

1) Cremer model for single panel;
2) Cremer model for double layer of panels;
3) TMM model with air layer, modelled as a porous layer;
4) laboratory measurement.

The measurements showed that the critical frequencies of the tested structures with 9.5 mm, 12.5 mm and 15 mm boards are 4000 Hz, 3150 Hz and 2500 Hz, respectively.

From the simulation for a single gypsum board performed with Cremer's theory, it can be observed that the critical frequency of two identical layers spaced by a thin air gap is indeed the same as that of a single board, whereas the critical frequency of a monolithic double thickness layer would have been lower.

The transfer matrix model appears to predict correctly both the position in frequency of the coincidence dip and the values of sound reduction index around and above the critical frequency. This indicates that the loss factor has been modelled correctly, which is also confirmed by the slope that the sound reduction index curve features beyond the critical frequency.

It is important to note that if a classic transfer matrix approach for the air layer between the panels is used, a considerable underestimation of the performance around the cavity resonance frequency is obtained.

Below 400 Hz, the agreement of the simulations with the measured values becomes poor. This is due to the fact that, in the gypsum board transfer matrix, the wall impedance Z_P has been estimated through Cremer's theory, which is valid for infinite panels. The discrepancy between the curves is therefore due to the modal behaviour of the actual panel and to the so-called "baffle effect".

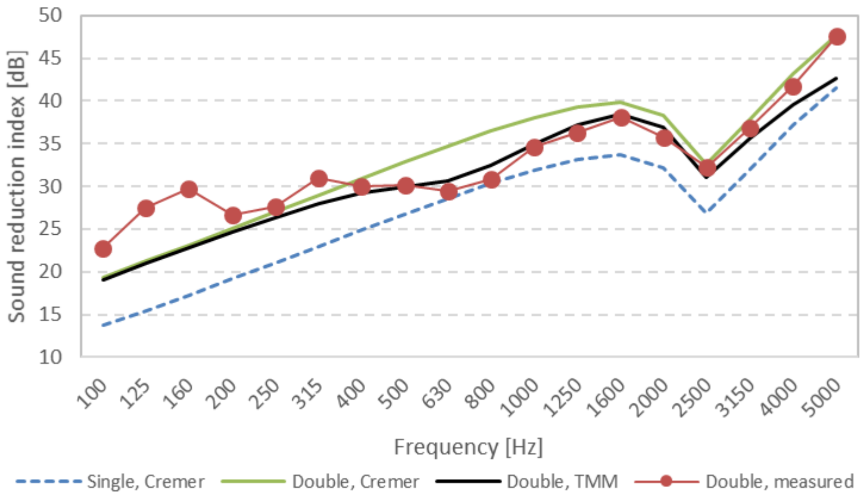

Fig. 3 – Double gypsum board 9.5 mm + 9.5 mm model comparison

Fig. 4 – Double gypsum board 12.5 mm + 12.5 mm model comparison

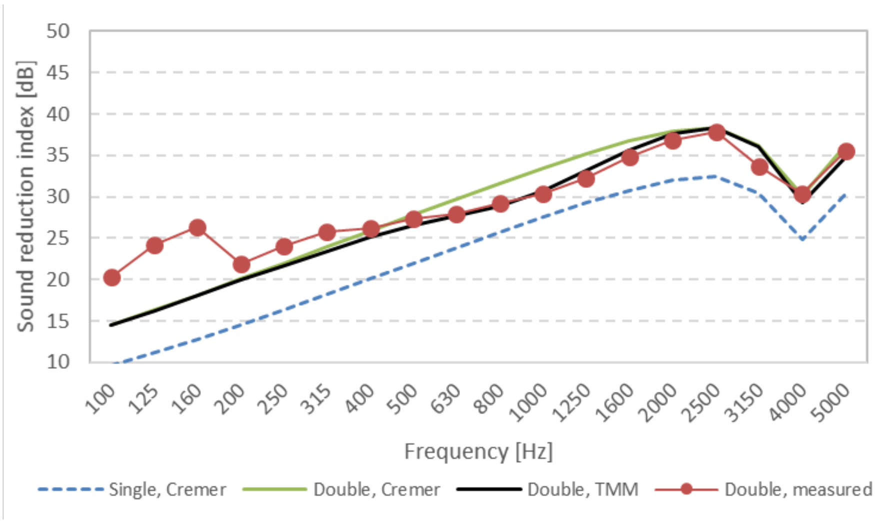

Fig. 5 – Double gypsum board 15 mm + 15 mm model comparison

4. Conclusion

This paper has studied the acoustic insulation performance of double layer gypsum board panels separated by a thin air gap. The multilayer acoustic structure was modelled with the transfer matrix method. In particular, the thin air layer was represented as a porous layer with very low flow resistivity, whereas the transfer matrix of the gypsum panel was built by using Cremer's theory for infinite homogenous plates. The results of the simulations were compared with measurements in sound transmission suites. The modelling technique proved to be suitable to estimate correctly some fundamental elements of the acoustic behaviour of the structure, such as the maintenance of the same critical frequency as the single gypsum layer, the values of the sound reduction index and the slope of the curve at and above the coincidence region. At low frequencies, where the modal behaviour of the structure dominates, a discrepancy is observed between simulations and measurements, arguably due to the finite size of the tested panel with respect to the infinite panel theory used to estimate the wall impedance in the panel's transfer matrix.

Nomenclature

Symbols

τ	sound transmission coefficient (-)
W	sound power (W)
R	sound reduction index (dB)
L	mean equivalent sound pressure level (dB)
S	surface of the specimen
A	equivalent absorption area of the receiving room (m^2)
V	volume of the receiving room (m^3)
t_r	mean reverberation time of the receiving room (s)
μ	mass per unit area (kg/m^2)
t	thickness of the panel (m)
ρ	density (kg/m^3)
E	Young's modulus (GPa)
ν	Poisson's modulus
η	loss factor
f	frequency (Hz)
f_c	critical frequency (Hz)
T	transfer matrix
L	thickness of the air layer (m)
L_a	thickness of the porous layer (m)
θ	angle of incidence in air (rad)
θ_a	angle of incidence in a porous layer (rad)
Z_p	impedance of single gypsum-panel (Pa s m^{-1})
Z_a	impedance of porous layer (Pa s m^{-1})
Γ_a	porous complex propagation coefficient (m^{-1})
Z_0	impedance of air (Pa s m^{-1})
Γ_0	air complex propagation coefficient (m^{-1})
k_0	wave number (m^{-1})
ω	angular velocity (rad/s)
D	bending stiffness (m^{-1})
σ	air flow resistivity (Pa s/m^2)

Subscripts/Superscripts

1	transmitting room
2	receiving room
w	weighted value

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