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SET OPTIMIZATION meets FINANCE II

Second International Conference on Set-Valued Variational Analysis and Optimization with Applications in Finance

Free University of Bozen-Bolzano in Bruneck-Brunico, Italy

September 8-12, 2014

unibz

CONTENTS

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LOCATION

Free University of Bolzano
Brunico Campus
Piazzetta dell'Universitá 1
I-39031 Brunico

DEAR PARTICIPANT,

this symposium is devoted to set-valued optimization, the underlying convex and variational analysis for vector- and set-valued functions and applications to financial market models with transaction costs. The aim is to give you an opportunity to present your results and discuss your points of view to set-valued optimization including equilibrium problems, variational inequalities, financial models using set-valued functions, scalarization procedures and algorithmic methods for such models.

We wish you a fruitful and stimulating time at the workshop, and hope you will enjoy your stay in Bruneck! If you have any questions, please don't hesitate to contact us.

The organizing commitee

VENUE | TRANSPORT

Bruneck-Brunico is the largest town in the Puster Valley (Pustertal-Valle de Pusteria), a world-class touristic spot in summer and winter as well as – probably less well known – a manufacturing center as represented by the supply industry companies GKN Driveline and GKN Sinter Metals. The Free University of Bozen maintains a Bachelor Program in Tourism, Sports and Event Management in Bruneck.

The university building at Universitätsplatz 1 - Piazzetta dell'Universitá 1 is located within the pedestrian area next to one of the four medieval age town gates, and almost everything in Bruneck can be reached by a 5 minute walk. This includes a variety of restaurants and RIPA, one of the Messner Mountain Museums in the old Bishop's castle of Bruneck.

The famous ski and hiking area Kronplatz-Plan de Corones at 2275m elevation can be reached via cable car departing from Reischach-Riscone, a 10 minutes city bus ride from Bruneck. The Tourist Information is located at Rathausplatz - Plaza Municipio, a 2 min walk from the university building. The Bus Station is at Europastrasse - Via Europa, a 5 min walk and the Railway Station on the same street a little further down.

A city map and a timetable for the citybus lines 1-3 can be found in the conference material.

In emergencies, please don't hesitate to call (0039) 333 375 1869.

Names of towns and villages usually come in pairs in South Tyrol. For example, Bolzano and Brunico are the Italian versions of Bozen and Bruneck, respectively, so don't get confused. This also applies to street names: for example, Stadtgasse is the German name of Via Centrale, the central pedestrian area of Bruneck - Brunico. You may try to use the Italian (German) version of a town/street name if the German (Italian) didn't work while looking something up on mapquest, googlemaps or openstreetmap.

WELCOME RECEPTION

On Monday, September 8, there will be a Welcome Reception which is at the same time the Vernissage of the Exhibition “Geometry in Fire," a photo contest for pictures with geometrical motives made of fire. This event will take place in the historical Ragenhaus, a 10 min walk along the Stadtgasse - Via Centrale. The meeting point for those who want to have a guided tour is 7:30pm in front of the university building.

INTERNET

The net openAiR is available everywhere in the university building free of charge. A password is not required.

CONFERENCE EQUIPMENT

The conference room is equipped with a **projector** and a **chalkboard**. A computer with a PDF and a POWERPOINT viewer is available. Please bring your presentation on a USB memory stick and transfer it before your session.

Speakers whose presentations contain animations or videos, or require a specific version of the viewer software, are strongly encouraged to use their own notebook and to test the system well in advance.

AWARDS

There will be a book award for the best young researcher’s presentation sponsored by Springer-Verlag.

SOCIAL PROGRAM

Wednesday: There will be a hiking tour to the famous Plan de Corones - Kronplatz at 2275m elevation. We will go up by cable car and then do round trips in two groups: one for experienced hikers and a more relaxing tour. In any case bring solid shoes and weather protection. The bus transfer to Riscone - Reischach is provided, but the tickets for cable car are not included (approx. 15 Euros for a return ticket).

Thursday: Conference dinner at the mountain restaurant Amaten. The bus transfer is provided. Please, if you didn't register yet, but want to take part, register until Tuesday 3pm at the registration desk.

SCHEDULE | MONDAY, SEPTEMBER 8, 2014

2:00pm–4:00	**B. Rudloff, Z. Feinstein**	Tutorial: Set-Valued Models in Finance
	Coffee break	
4:30–6:30	**A. Löhne, B. Weißing**	Tutorial: Computational Issues in Set Optimization
8:00	**Welcome reception at Ragenhaus Bruneck with vernissage "The Geometry of Fire"**	

SCHEDULE | TUESDAY, SEPTEMBER 9, 2014

9:00am	**Opening remarks: Prof. Corteau, Prof. Bergmeister, Prof. Maurer** **Moderation: A. Hamel**	
9:30–10:15	**Keynote by R.T. Rockafellar**	Risk measures and utility: linkage and extensions
	Coffee break	
10:45–11:15	**D. Kuroiwa**	Robust vector optimization by set optimization
11:20–11:50	**M. Rocca**	Quasiconvexity of set-valued maps assures well-posedness of robust vector optimization
11:55–12.25	**R. Strugariu**	On set-valued optimization problems with variable ordering structure
	Lunch break	
2.30–3.15	**Special guest P. Koch-Medina**	Risk measures with respect to multiple eligible assets
3:20–3.50	**C. Munari (young researcher)**	Risk measures with multiple eligible assets (II)
	Coffee break	
4:20–4:50	**L. Csirmaz**	Optimal solutions of an optimization problem defined by an oracle
4:55–5:25	**M. Ehrgott**	From set optimization to data envelopment analysis - a tale of sets, extreme points, hyperplanes, and money
5.30–6:00	**F. Bökler (young researcher)**	Running time analyses of Benson type algorithms with an application to multiobjective combinatorial optimization problems
	Time for dinner, night off.	

SCHEDULE | WEDNESDAY, SEPTEMBER 10, 2014

8:30-9:15	**Keynote by I. Molchanov**	Set-valued portfolios and set-valued risks
9:20–9:50	**C. Zalinescu**	Series of convex functions in locally convex spaces: subdifferential, conjugate and applications
	Coffee break	
10:20–10:50	**A. Roux**	Hedging and optimal exercise of American options with gradual exercise under transaction costs
10:55–11:25	**F. Heyde**	Geometric duality for convex vector optimization problems
11:30–12:00	**F. Ulus (young researcher)**	Parametric simplex algorithm for linear vector optimization problems
12:00–1:00	**Time for lunch**	

1:00–7:00	**Hiking tours (Kronplatz, two different tours)**

SCHEDULE | THURSDAY, SEPTEMBER 11, 2014

9:00–9:45	**Keynote by G. Crespi**	Refreshing variational inequalities in set optimization
9:50–10:20	**C. Schrage**	A Weierstrass extreme value theorem in set optimization
	Coffee break	
10:50–11:20	**M. Pilecka (young researcher)**	Optimality conditions in set-valued programming
11:25–11:55	**M. Studniarski**	Necessary and sufficient conditions for a Pareto optimal allocation in a discontinuous Gale economic model
12:00–12:30	**G. Panda**	Bounds of performance measure of a portfolio optimization model with set valued parameters
	Lunch break	
2.30–3.15	**M. Karliczek (young researcher)**	Elements of L^0-module theory
3:20–3:50	**S. Cerreia-Vioglio**	Hilbert A-Modules
	Coffee break	
4:20–4:50	**C. Ararat (young researcher)**	A characterization theorem for Aumann integrals using the complete lattice approach
4:55–5:25	**Z. Feinstein**	A set-valued Bellman's principle
7.00	**Departure for Conference Dinner (bus transfer to Amaten)**	
	approx. 10:30-11pm bus back from Amaten	

SCHEDULE | FRIDAY, SEPTEMBER 12, 2014

9:00–9:45	**Keynote by T. Pennanen**	Optimal investment and contingent claim valuation in illiquid markets
9:50–10:20	**N. Popovici**	On certain classes of generalized convex set-valued functions
10:20–10:35	**Award session**	
	Coffee break	
11:05–11:50	**Special guest B. Mordukhovich**	Applications of set optimizaion to behavioral sciences
11:55–12:25	**I. Sadeqi**	Lipschitz continuity of an approximate solution mapping for set-valued vector equilibrium
12:30–1:00	**B. Rudloff**	Set-valued measures of multivariate and systemic risk
1:00–1:05	**Closing remarks**	

ABSTRACTS

Tutorial: Set-valued Models in Finance

Zach Feinstein and Birgit Rudloff, Princeton University

Abstract

We will present an overview over recent research in finance involving set-valued models. Topics include

- multivariate risks and set-valued risk measures, which are useful if market frictions like illiquidity or transaction costs are present,
- systemic risk measures for the banking sector,
- incomplete preference relations and related utility maximization problems
- price bounds in markets with frictions.

Special attention will be given to motivating the use of set- and vector-valued constructions in finance. Computational issues are briefly discussed which relate the set-valued approach to vector optimization.

Tutorial: Computational Issues in Set Optimization

Andreas Löhne and Benjamin Weißing, MLU Halle-Wittenberg

Abstract

This tutorial is aimed at providing an introduction to numerical methods for set optimization problems, prior knowledge of this topic is no prerequisite. In the first part, we summarize different approaches to set optimization and related solution concepts. The second part is devoted to numerical methods for vector optimization problems, an important special case of set optimization. In particular, we consider different tools for solving vector optimization problems. The third part deals with algorithms for solving set optimization problems (of which rather few exist hitherto) beyond the special case of vector optimization.

A characterization theorem for Aumann integrals using the complete lattice approach

Cagin Ararat, Princeton University

Abstract

The Aumann integral of a measurable function that maps into the power set of $\mathbb{R}^m$ is defined as the set of all vector Lebesgue integrals of its integrable selections. Using the complete lattice approach, set-valued functions whose values are closed convex upper sets are considered and it is shown that their Aumann integrals are also closed convex upper sets. The main result is a Daniell-Stone type characterization theorem for Aumann integrals of such functions. More precisely, the result characterizes the conditions under which a functional that maps from a certain collection of measurable functions into the set of all closed convex upper sets can be written as the Aumann integral with respect to some σ-finite measure. These conditions include the analog of the (con)linearity and monotone convergence properties of the classical Lebesgue integral, a property that ensures σ-finiteness of the measure and two properties that are peculiar to the set-valued case as they are redundant in the one-dimensional setting.

Running Time Analyses of Benson Type Algorithms with an Application to Multiobjective Combinatorial Optimization Problems

Fritz Bökler, TU Dortmund

Abstract

Although multiobjective optimization often appears in practical applications, it is usually regarded as too expensive to pursue. Albeit, theoretical running time guarantees are seldom considered in the literature. In this talk, we present the first running time analyses of Benson's "outer approximation algorithm" and its dual variant. We show that they run in polynomial total time, i.e., polynomial in the input and the output size, for a fixed but arbitrary number of objectives with respect to a set of LP oracle problems. Moreover, we suggest improvements to remove an exponential running time burden from both algorithms.

Using the above results, we prove that if the single objective optimization variant of a multiobjective combinatorial optimization problem can be solved in strongly polynomial time then enumerating the extreme nondominated points of the Pareto-frontier can be done in polynomial total time for every fixed number of objectives. Since this was only known for biobjective problems, our results close the gap between these problems and problems with three and more objectives. Additionally, we show that this can be accelerated by using our suggested improvements.

This enables us to present—to the best of our knowledge—the first computational study for enumerating the extreme nondominated value vectors of the multiobjective assignment and spanning tree problem with more than four objectives. The results show that this algorithm is very competitive compared to the small number of algorithms available to solve this problem for three and four objectives.

Hilbert A-Modules

Simone Cerreia-Vioglio, Universitá Bocconi

joint work with *Fabio Angelo Maccheroni,* and *Massimo Marinacci*

Abstract

We consider pre-Hilbert modules H on Archimedean f-algebras A with unit e. We provide conditions on A and H such that a Riesz representation theorem for bounded/continuous A-linear operators holds.

Refreshing variational inequalities in set optimization

Giovanni P. Crespi, University of Valle d'Aosta

Abstract

Since the seminal papers by Stampacchia and Minty, a big hit in variational inequality theory has been the study of the relations with a primitive optimization problem. Indeed, the so called differentiable variational inequalities provide necessary and sufficient optimality conditions for scalar optimization under mild assumptions. Continuity and convexity, mostly, plays the role of key hypothesis to prove these results. In the last decades, some generalization of the classical variational inequality has been proposed to consider non differentiable optimization problems. Indeed, by means of a Dini type derivative, it has been proved that the existence of a solution to the Minty variational inequality entails some regularity on the primitive optimization problem. Some attempt to extend these results to vector optimization has been undertaken as well. However, mainly due to the lack of a complete order in the image space, the results proved are not exactly matching those for the scalar counterpart. As for the case of set-valued objective functions, only the recent introduction of set-optimization theory has allowed to overcome most of the difficulties that were foreseeable when dealing with set-valued objective function. In this talk we present a new approach to construct a variational inequality for set-valued optimization problems, based on the set-optimization approach. Providing a reasonable solution concept, we can extend to set-optimization some results known for the scalar case. Results for vector optimization can be obtained as an application of those in set-optimization, legitimating the latter as a desirable setting to study the former.

Optimal solutions of an optimization problem defined by an oracle

Laszlo Csirmaz, Central European University, Budapest

Abstract

Finding optimal, or almost optimal points of a linearly constrained convex region is a problem with important applications in finance, business, medicine, physics, and even in theoretical parts of mathematics. Frequently there is no clear objective of the optimization, rather several contradicting goals should be taken into account. Solving such linear multiobjective optimization problems *efficiently* is the main focus of several recent research works. Our contribution to this research:

1. taking a bird's-eye view of the multi-objective optimization prob- lems and the proposed algorithms for their solution, we describe an abstract problem which discards particular details and which lets us to see what the "big picture" is;
2. show that Benson's algorithm actually solves the problem identified above;
3. propose another algorithm which avoids the uncontrollable blow-up caused by the double description method.

From Set Optimization to Data Envelopment Analysis - A Tale of Sets, Extreme Points, Hyperplanes, and Money

Matthias Ehrgott, University of Auckland

Abstract

Set Optimization provides a new theoretical foundation of multiobjective linear programming that underlies the primal and dual versions of Benson's outer approximation algorithm to solve multiobjective linear programs. These algorithms provide a new perspective on the performance analysis tool of data envelopment analysis. Resulting algorithms are much more efficient than standard DEA approaches, and therefore make this tool, that is widely used in the finance sector, much more applicable in practice.

A Set-Valued Bellman's Principle

Zach Feinstein, Princeton University

Abstract

A method for calculating multi-portfolio time consistent multivariate risk measures in discrete time is presented. Market models for d assets with transaction costs or illiquidity and possible trading constraints are considered on a finite probability space. The set of capital requirements at each time and state is calculated recursively backwards in time along the event tree. We motivate why the proposed procedure can be seen as a set-valued Bellman's principle. We give conditions under which the backwards calculation of the sets reduces to solving a sequence of linear, respectively convex vector optimization problems

Geometric Duality for Convex Vector Optimization Problems

Frank Heyde, Technical University Bergakademie Freiberg

Abstract

Geometric duality for linear vector optimization problems turned out to be useful as a basic tool for developping and improving Benson-type algorithms for solving linear vector optimization problems. In this lecture we will introduce a generalization of the theory to convex problems using a duality relation of epigraphs and generalized second order calculus.

Elements of L^0-Theory

Martin Karliczek, Humboldt University Berlin

joint work with *Bielecki, Cialenco, Drapeau, Hamel, Kupper* and *Streckfuß*

Abstract

For the set $L^0 = L^0(\Omega, \mathcal{F}, P)$ of all random variables on a probability space, which are identified if they coincide almost surely, we consider an arbitrary L^0-module. Cheridito, Filipović, Guo, Kupper and Vogelpoth were the first to consider L^0 as a substitute for the real numbers to handle L^0-modules. It turned out that in L^0-modules, the action of indicator functions of partitions on sets and functions is essential and the two main technical properties to demand when working in this setting are locality of functions and σ-stability of sets. We examine three different problems within the theory of L^0-modules: a Brouwer fixed point theorem, a robust representation of quasiconcave functions and an Ekeland's variational principle. Doing so we illustrate features of L^0-convex analysis and show that defining objects as the left-inverse of functions, premetrics or simplexes, we permanently have to pay attention to the properties of locality and σ-stability.

Risk measures with respect to multiple eligible assets

Pablo Koch-Medina, University of Zurich

Abstract

We study risk measures for financial positions representing the minimum amount of capital to raise and invest in portfolios of a set of pre-specified traded assets in order to meet a prescribed acceptability constraint. We investigate finiteness and continuity properties of these multi-asset risk measures, highlighting the interplay between the acceptance set and the class of eligible portfolios. We discuss an alternative approach to dual representations of convex multi-asset risk measures which relies on the external characterization of closed convex acceptance sets

Robust vector optimization by set optimization

Daishi Kuroiwa, Shimane University

Abstract

Mathematical programming problems with data uncertainty are very important due to the reality of uncertainty in real-world problems. Robust optimization, which has emerged as a powerful deterministic approach for studying mathematical programming with data uncertainty, associates an uncertain mathematical program with its robust counterpart.

In this talk, we consider vector optimization problems with data uncertainty and we observe some types of robust and optimistic solutions which are concerned with set optimization.

Keywords: robust optimization, vector optimization, set optimization.

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Set-valued portfolios and set-valued risks

Ilya Molchanov, University of Bern

Abstract

Using the concept of set-valued portfolios, it is possible to incorporate dependencies between the distribution of the multivariate gain and the exchange rules. The set-valued portfolio is considered acceptable if it possesses a selection (i.e. a random vector which almost surely belongs to it) with all individually acceptable components. The corresponding set-valued risk measure is said to be the selection risk measure. The talk surveys its main properties, discusses the primal and dual representations, gives examples, and shows how to approximate its values from below and from above using rather elementary arguments. Applications of selection risk measures to the practical risk assessment are also presented.

Applications of set optimizaion to behavioral sciences

Boris Mordukhovich, Wayne State University

joint work with *Truong Bao* and *Antoine Soubeyran*

Abstract

This talk concerns mathematical models arising in psychology and some other areas of behavioral sciences that are formalized via general preferences with variable ordering structures. Our considerations are based on variational principles and tools of set optimization via the recent "variational rationality approach" to behavioral science modeling suggested by Antoine Soubeyran. This approach unifies numerous theories in different branches of behavioral sciences by using, in particular, worthwhile change and stay dynamics and variational traps. In the mathematical framework of this approach, we derive a new variational principle, which can be viewed as an extension of the Ekeland variational principle to the case of set-valued mappings on quasimetric spaces with cone-valued ordering variable structures. Such a general setting is proved to be appropriate for broad applications to the functioning of goal systems in psychology and other areas. In this way we give a certain answer to the following striking question: in the world, where all things change (preferences, motivations, resistances, etc.), where goal systems drive a lot of entwined course pursuits between means and ends–what can stay fixed for a while? The obtained mathematical results and new insights open the door to developing powerful models of adaptive behavior, which strongly depart from pure static general equilibrium models of the Walrasian type that are typical in economics.

Risk measures with multiple eligible assets (II)

Cosimo-Andrea Munari, ETH Zürich

Abstract

We focus on dual representations for risk measures, both from a scalar and a set-valued perspective. In particular, we show how to use the robust representations for scalar risk measures studied in Farkas, Koch-Medina, Munari (2014) to obtain robust representations of set-valued risk measures as in Hamel, Heyde, Rudloff (2011). This may strengthen the link between these two approaches and provide additional hints how to extend known scalar representations (and their dual variables) to set-valued representations.

Bounds of performance measure of a portfolio optimization model with set valued parameters

Geetanjali Panda, Indian Institute of Technology, Kharagpur

Abstract

One important objective of selecting a portfolio, based on risk-adjusted performance, is to maximize the Sharpe ratio. The greater a portfolio's Sharpe ratio, the better its risk-adjusted performance has been. A negative Sharpe ratio indicates that a risk-less asset would perform better than the security/risky asset. Maximizing Sharp ratio optimization problem is usually solved using general fractional programming technique, where the parameters like return, variance etc. are estimated from historical data. However, in the financial market, several types of uncertainties are affecting while estimating these parameters. For this reason, it is reasonable to estimate the lower and upper bound of the expected return in stead of estimating the expected return directly. The advantage of estimating the bounds of the expected return is that these bounds can include all possible values corresponding to different uncertainties. In that case all the parameters like return, standard deviation, risk etc. will be sets, which are closed intervals. Subsequently, the Sharp ratio will be the ratio of two set valued functions. Hence the optimal Sharpe ratio lies in a set, instead of being a fixed real number. In other words, the user can attain the lower and upper bound of the maximum value of the Sharpe ratio corresponding to an optimal portfolio, where the upper bound represents the most acceptable value of Sharpe ratio and lower bound represents the value of Sharpe ratio which will be acceptable for the investors. But general fractional programming technique will not help to solve such a model because this optimization model has ratio of set valued functions in the objective and set inclusions in the constraints to compare two set valued mappings. This paper develops a solution methodology to address this difficulty. Here closeness between two set valued functions is defined using a function which maps two sets to [0,1], known as chi-acceptable function. Using the concept of chi-acceptable function the set optimization model is transformed to a deterministic non linear programming problem to obtain an efficient portfolio which provides the upper and lower bound of optimum Sharpe ratio. The theoretical development is illustrated in a portfolio selection model with historical data from the Indian Stock Market.

Optimal investment and contingent claim valuation in illiquid markets

T. Pennanen, King's College London, U.K.

Abstract

We extend basic results on arbitrage bounds and attainable claims to illiquid markets and general swap contracts where both claims and premiums may have multiple payout dates. Explicit consideration of swap contracts is essential in illiquid markets where the valuation of swaps cannot be reduced to the valuation of cumulative claims at maturity. We establish the existence of optimal trading strategies and the lower semicontinuity of the optimal value of optimal investment under conditions that extend the no-arbitrage condition in the classical linear market model. All results are derived with the "direct method" without resorting to duality arguments.

Optimality conditions in set-valued programming

Maria Pilecka, TU Bergakademie Freiberg

Abstract

We consider a set-valued optimization problem where a set-valued objective mapping is minimized over a feasible set given by a closed convex set. In this talk the notion of optimality introduced by Kuroiwa is regarded. The images of the set-valued mapping are assumed to be compact and convex whereas the mapping is not convex. Such kind of optimization problem may be interpreted as a bilevel optimization problem with a convex lower level problem possessing a compact feasible set.

After introducing a special set difference, we define a directional derivative and a subdifferential for set-valued mappings and investigate properties of these tools. We derive new optimality conditions for unrestricted set-valued optimization problems using both directional derivative and subdifferential. In addition, we also present optimality conditions for restricted problems with the aid of the tangent cone (Bouligand cone) to the feasible set and the directional derivative.

On certain classes of generalized convex set-valued functions

Nicolae Popovici, Babes-Bolyai University

Abstract

We study several classes of generalized convex set-valued functions, which are currently used in vector/set optimization. In particular, we present some recent results, jointly obtained with Daishi Kuroiwa and Matteo Rocca, concerning the characterization of cone-convex set-valued functions by means of cone-quasiconvexity.

Quasiconvexity of set-valued maps assures well-posedness of robust vector optimization

Matteo Rocca, University of Insubria

joint work with *Giovanni P. Crespi,* and *Daishi Kuroiwa*

Abstract

Robust optimization is a fast growing methodology to study optimization problems with uncertain data. An uncertain vector optimization problem can be studied through its robust or optimistic counterpart. In this paper we formulate the counterparts as set optimization problems, that appears to be a more natural framework, especially when the uncertain problem is a non linear vector optimization problem.

Under this setting we study the well-posedness of both the robust and the optimistic counterparts, using the embedding technique for set optimization developed by Kuroiwa. To prove our main results we also need to study the notion of quasiconvexity for set-valued maps, that is the property of convexity of level sets. In the paper we provide a general scheme, based on ordering relations, to define the notion of level sets and we study the relations among different definitions of quasi-convexity. Existing notions prove to be special cases of our scheme.

Risk Measures and Utility: Linkage and Extensions

R. Tyrell Rockafellar, University of Washington

Abstract

Maximizing the expected utility of a random variable representing profit or gain is a widely used approach to financial decision-making. Alternatively, portfolios can be put together to minimize risk as captured by the choice of a measure of risk as a functional applied to random variables representing costs or losses. Some connections are known between the two approaches, but there is a deeper linkage, not yet fully appreciated, in which a measure of risk can very broadly be portrayed as coming from trade-off rules with respect to "utility" as a functional on a space of random variables. That requires extending beyond just expectations and on the other hand considering utility to be relative to some benchmark. Such extension opens remarkable connections between utility and statistical analysis using generalized regression tuned to particular types of risk.

A different kind of extension, for risk measures directly, is to articulate them for vector-valued random variables. Surprisingly, this really leads nowhere. Under typical assumptions that make sense for risk measures, akin to coherency, such extented functionals turn out inevitably to exhibit separability and reduce to expressions coming merely from a vector of scalar-valued risk measures, one for each scalar component of the vector random variable.

Hedging and optimal exercise of American options with gradual exercise under transaction costs

Alet Roux, University of York

joint work with *Tomasz Zastawniak*

Abstract

I will present algorithms based on linear vector optimization methods to construct sets of superhedging strategies for both the buyer and seller of an American option in the presence of proportional transaction costs. Allowing the buyer to exercise the option gradually (rather than instantly as is usually assumed in the literature) means that the set of superhedging strategies for the buyer becomes convex, and linear vector optimization methods can be used. The algorithm for the seller is a direct adaptation of a known algorithm for European options. These algorithms enable the efficient construction of optimal hedging strategies and exercise times for both the buyer and the seller.

Set-valued measures of multivariate and systemic risk

Birgit Rudloff, Princeton University

Abstract

Set-valued risk measures have been used in markets with illiquidities. In this talk we will show that another application is the measurement of systemic risk, that is, the risk that a network of interconnected banks imposes to the outside economy. We model the financial system via a network of obligations as introduced by Eisenberg and Noe (2001), but with random endowments for each firm. The systemic risk measure is defined as the collection of vectors of capital requirements of the banks that make the impact to the outside economy acceptable. Properties of the risk measure are deduced, various allocation mechanisms are discussed, and numerical case studies with differing network structures and stress scenarios are given.

Lipschitz Continuity of an Approximate Solution Mapping for Set-Valued Vector Equilibrium

I. Sadeqi, Sahand University of Technology

joint work with *M. Salehi*

Abstract

The purpose of this paper is to generalize and improve the connectedness and the compactness of solutions set for set-valued vector equilibrium problems by using the scalar characterization method. Moreover, the Lipschitz continuity of an approximate solution mapping for the parametric set-valued vector equilibrium problems is studied.

A Weierstrass extreme value theorem in set optimization

Carola Schrage, University of Valle d'Aosta

joint work with *Giovanni P. Crespi, Andreas H. Hamel,* and *Matteo Rocca*

Abstract

In set-optimization, a solution to a extreme value theorem is considered to be a infimizer, i.e. a set in the pre-image space over which the infimum of the given function is attained, consisting of elements whose images satisfy a certain minimality condition. We will prove that under a compactness assumption on the level sets and lower continuity of a given set of scalarizations, such a solution always exists. This result is in line with the well known scalar version of the Weierstrass extreme value theorem.

On Set-Valued Optimization Problems with Variable Ordering Structure

Radu Strugariu, "Gheorghe Asachi" Technical University of Iaşi

Abstract

We introduce and investigate an optimality concept for set-valued optimization problems with variable ordering structure. In our approach, the ordering structure is governed by a set-valued map acting between the same spaces as the objective multifunction. Necessary optimality conditions for the proposed problem are derived in terms of Bouligand and Mordukhovich generalized differentiation objects.

Necessary and sufficient conditions for a Pareto optimal allocation in a discontinuous Gale economic model

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joint work with *Anna Michalak*

Abstract

In some economic models (see, for example, [1], [3]) one has to consider discontinuous functions. The existing research results on such models are focused on the existence of equilibria. The aim of this paper is to present necessary and sufficient conditions for local Pareto optimality in the Gale model involving possibly discontinuous functions.

We now describe a simplified version of the Gale model [1]. Suppose we have n goods $G_1, ..., G_n$ and p economic agents $A_1, ..., A_p$. The set of goods includes all types of labor and services as well as material commodities. The economic agents may be thought of as either consumers or as producers.

The amount of goods $G_1, ..., G_n$ supplied or consumed by an agent A_i certain fixed time interval is given by a vector

$$x_i = (x_{i,1}, ..., x_{i,n}) \in R^n. \tag{1}$$

The j-th coordinate $x_{i,j}$ represents the amount of the good G_j and is positive (respectively, negative) if G_j is supplied (respectively, consumed). Such a vector is called a *commodity bundle* of A_i. The set C_i of all possible commodity bundles (1) is called the *commodity set* or *technology set* of the agent A_i, $i = 1, ..., p$.

In the Gale model it is assumed that the *balance inequalities* hold, i.e., the total amount of each good consumed by all agents must not exceed the total amount supplied:

$$\sum_{i=1}^{p} x_{i,j} \geq 0, \quad j = 1, ..., n \tag{2}$$

Definition 1*A vector system* $\{x_1, ..., x_p\}$ *is called a* feasible allocation *if* $x_i \in C_i$, $i = 1, ..., p$, *and inequalities* (2) *hold.*

Let us note that condition (2) may be written down in an equivalent vector form

$$\sum_{i=1}^{p} x_i \geq 0.$$

We assume that, for each agent A_i, there exists a utility function $h_i : C_i \to \mathbb{R}$, $i = 1, ..., p$. Each agent tends to maximize his utility function.

Definition 2*A feasible allocation* $\{fx_1, ..., x_p\}$ *is called a* Pareto optimal allocation *if, for every other feasible allocation* $\{x_1, ..., x_p\}$, *we have either*

$$h_i(x_i) = h_i(\overline{x}_i) \text{ for all } i \in \{1, ..., p\} \tag{3}$$

or

$$h_j(x_j) < h_j(x_j) \text{ for some } j \in \{1, ..., p\}. \tag{4}$$

In this paper, we obtain higher-order necessary and sufficient conditions for a locally Pareto optimal allocation (a local version of Definition 2). These conditions are obtained by applying the recent results of [2]and are formulated in terms of generalized lower and upper directional derivatives of utility functions.

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Parametric Simplex Algorithm for Linear Vector Optimization Problems

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Abstract

We propose a parametric simplex algorithm for solving linear vector optimization problems (LVOPs). It is a generalization of the parametric self-dual simplex algorithm, which originally is designed for solving single objective linear optimization problems, and capable of solving two objective LVOPs whenever the ordering cone is the positive orthant. Our algorithm works for any dimension, and it is possible to extent it to any polyhedral ordering cone C. In each iteration, the algorithm provides a set of inequalities, which defines the current partition of the parameter space and correspond to a vertex of the upper image. In addition to the usual simplex arguments, one needs to eliminate the redundant inequalities from that set. This extra step is similar to the vertex enumeration procedure, which is used in most of the objective space based LVOP algorithms. Different from those, this algorithm doesn't require to solve a scalar linear program in each iteration.

Series of convex functions in locally convex spaces: subdifferential, conjugate and applications

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Abstract

X. Y. Zheng, in [1], studied the subdifferential of the sum of a series of convex functions defined on a Banach space. In this note we extend Zheng's results to series of convex functions defined on locally convex spaces and apply them for calculating the conjugate of the sum. Then we apply the obtained results for giving rigorous proofs for the Maxwell–Boltzmann, Fermi–Dirac and Bose–Einstein entropies.

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