Should unemployment benefits be related to previous earnings?

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Abstract:
In most OECD countries, unemployment benefits are tied to individual previous labor earnings. We study the progressivity of this indexation with regard to its effects on employment, output, and welfare in a calibrated general equilibrium model with search unemployment. Employment varies endogenously on both the intensive margin and the extensive margins as agents choose their labor supply (if employed) or their search effort (if unemployed) in order to optimize life-time utility. Compared to the case of lump-sum unemployment compensation, a system of insurance payments that are related to past contributions results in higher output and welfare. The effects on employment are negligible as employed workers rather accumulate more savings than to supply additional work in order to insure against the loss of employment.

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1 Introduction

Unemployment insurance (UI) schemes are a distinctive feature of modern economies and have been frequently recognized to play an important role in determining labor market outcomes. The possible influence of the level and duration of unemployment benefits on unemployment and welfare has been given considerable attention in research. The general argument usually put forward is that unemployment benefits improve the pay-off from not working and decrease the incentives to supply labor. Accordingly, recent work on the employment effects of unemployment benefits emphasizes the moral hazard associated with the job search effort of the unemployed, e.g. Hansen/İmrohoroğlu (1992) or Ljungqvist/Sargent (1998), the moral hazard associated with the job retention effort, e.g. Wang/Williamson (1996), and the direction of search effort to high wage jobs, e.g. Burdett (1979), Acemoglu (1997), Acemoglu/Shimer (1997) or Marimon/Zilibotti (1997). In addition, Shavell/Weiss (1979), Frederiksson/Holmlund (2001), and Heer (2002,2003) emphasize the fact that the duration period of most unemployment insurance programs are limited and find that UI benefits should, optimally, decline over time.

The literature discussed in the preceding paragraph, however, considers the level of benefits to be lump-sum. In most OECD countries, unemployment benefits consist of both unemployment insurance and unemployment assistance.\footnote{For the features of the different UI schemes in practice see OECD (1991, 1996).} Most often, as e.g. in Belgium, France, Germany or the US, unemployment insurance payments are related to past contributions and compensate for a loss of income for a limited duration. Afterwards, unemployed agents rely on unemployment assistance which is usually lower than unemployment insurance and most often unrelated to past contributions, e.g. in France or the UK.\footnote{As one of the few exceptions, German unemployment assistance payments are tied to previous contributions.} Furthermore, most countries provide a minimum unemployment income (often in the form of social assistance or welfare payments) and unemployment benefits are only paid up to a certain level, e.g. in Belgium, Spain, or Japan. Consequently, unemployment benefits in practice contain both...
a lump-sum component and a component that is proportional to previous unemployment contributions.

In our model, unemployment benefits depend on previous labor earnings. Therefore, the worker also considers the effect of his working hours on potential future unemployment benefits when he chooses his labor supply. If unemployed, agents search for a new job. Again, search effort is chosen in order to maximize life-time utility so that employment variation on both the intensive and the extensive margins is endogenous.\textsuperscript{3} If there is a successful match of a vacancy with a searching worker, wages result from collective bargaining. We further endogenize the financing of the unemployment insurance payments. In particular, we consider the case that total government expenditures on unemployment compensation are constant for all cases considered and are to be financed by unemployment insurance contributions. As a consequence, an increase of the part of unemployment insurance benefits that is proportional to past earnings results in a decrease of the lump-sum part of unemployment insurance benefits that is unrelated to previous earnings.

There are multiple effects of a more progressive indexation of unemployment benefits to previous earnings on equilibrium values of employment, output, and welfare: i) workers supply more labor in order to insure against the bad luck of unemployment and, in case of job separation, to be entitled to higher benefits. ii) Wages that result from Nash bargaining decrease and, consequently, firms increase their vacancy posting.\textsuperscript{4} iii) With increasing age, working agents have accumulated relatively high wealth and the supply of more labor is related to a higher loss of utility; therefore, agents may rather increase savings than labor supply in order to insure against the risk of unemployment. The first effect results in an increase of the average working hours. The second effect results in increase of the equilibrium number of employed agents. And the third effect results in higher capital

\textsuperscript{3}As one of the very few papers known to us, Cooley and Quadrini (1999) also model employment endogenously along both the intensive and the extensive margin. Contrary to us, however, Cooley and Quadrini assume households to have linear utility in consumption and to hold equal wealth.

\textsuperscript{4}This is only the partial equilibrium effect. In general equilibrium, wages may respond differently as discussed in more detail in section 3.
accumulation and higher output. The overall effect on aggregate effective employment (total hours worked) and output cannot be determined analytically. For this reason, we use a general equilibrium model that is calibrated with regard to the characteristics of the German economy in 1991-97.

The paper is organized as follows. Section 2 introduces the model and discusses our calibration. The derivation of the wage equation and the computation are described in the appendix. Our results are presented in section 3. Section 4 concludes.

2 The Model

The life-cycle model with endogenous unemployment is based on the general equilibrium models of Costain (1997). Three sectors can be depicted: the household sector, the production sector, and the government. Households live for 60 years and maximize discounted life-time utility. Agents can either be employed or unemployed during their working life. If employed, workers supply labor elastically. If unemployed, agents search for a job with utility-maximizing effort. Firms maximize the discounted sum of profits. They post vacancies, hire labor and capital, and pay out dividends to their shareholders. The government provides social insurance which it finances by a tax on wage income (or, equally, unemployment insurance contributions). Since we will only analyze steady-state allocations, the time index is omitted from stationary variables like, e.g., from the interest rate $r$ or the wage rate $w$.

2.1 Households

Agents live for $T + T^R = 60$ periods (60 years). The first $T = 40$ periods (40 years), they are workers. They are either employed supplying labor $l$ at the wage rate $w$, which is taxed at the rate $\tau$, or they are unemployed and search for a job with intensity $s$. The last $T^R = 20$ periods of their life, they retire and receive pension payments $w_R$. Households are
of measure one and each generation is of equal measure $1/60$. The household maximizes his intertemporal utility:

$$E \sum_{j=1}^{T+T^R} \beta^{j-1} \left[ \frac{c_j^{1-\sigma} - 1}{1 - \sigma} - \frac{(l_j + s_j)^\gamma}{\gamma} \right],$$

(1)

where $c_j$, $l_j$, and $E$ denote the consumption and the labor supply of the $j$-period old and the expectation operator conditional on information at the beginning of age $j$, which contains the agent’s employment status $\epsilon$, his wealth $a$ and his labor supply $l^+$ during his last period of employment. Instantaneous utility is discounted with the factor $\beta$, and $\sigma$ denotes the coefficient of relative risk aversion. Working or searching causes disutility $(l + s)^\gamma/\gamma$ to the agent.

The employment status $\epsilon$ can take four different values $\epsilon \in \{1, 2, 3, 4\}$: $\epsilon = 1$) agents are employed and receive after-tax wage $(1 - \tau)w_l$, $\epsilon = 2$) agents are unemployed and were employed in one of the previous periods and, hence, are entitled to unemployment insurance $w_{UI}$, $\epsilon = 3$) workers who have never found employment during their life have to rely on welfare payments $w_W$, and $\epsilon = 4$) retired agents who receive pensions $w_R$. If agents are employed or retired, they do not search ($s_j = 0$ for $\epsilon_j = 1$ or $j > T$).

Agents are born without any assets. Furthermore, agents face a borrowing constraint, $a_j \geq 0$. Furthermore, we assume that private insurance markets are absent.\(^5\) Depending on his employment status $\epsilon$ and his previous labor supply $l^+$, an agent at age $j$ receives labor income $y(\epsilon_j, l^+_j)$ and earns interest income at rate $r$:

$$a_{j+1} + c_j = (1 + r)a_j + y(\epsilon_j, l^+_j) + tr,$$

(2)

where $a_{j+1}$ and $tr$ denote next period’s asset holdings and government transfers, respectively. The labor income $y(\epsilon, l^+)$ of a household with employment status $\epsilon$ and previous labor supply $l^+$ is given by:

\(^5\)Chiu and Karni (1998) show that the presence of private information about individual’s work effort helps to explain the failure of the private sector to provide unemployment insurance.
\[
y(\epsilon, l^+) = \begin{cases} 
(1 - \tau)wl & \epsilon = 1 \\
w_{UI}(l^+) = w_{\min} + \theta(1 - \tau)wl^+ & \epsilon = 2 \\
w_W & \epsilon = 3 \\
w_R & \epsilon = 4.
\end{cases}
\]

The probability of employment in the first period is given by \( p_0 \). During their working life, agents lose employment with an exogenous probability \( \delta \). Unemployed agents looking for a job with search intensity \( s \) will find a job with probability \( 1 - e^{-\pi s^2} \), \( z \in (0, 1) \). For the individual agent, the probability parameter \( \pi \) and the probability of finding a job in the first period of life, \( p_0 \), are exogenous while they are determined endogenously in the labor market. The parameter \( z \) influences the response of the individual search effort to a change in policy. For low values of \( \pi \) and \( s \), the individual probability to find a job is approximately equal to \( \pi s^2 \) and \( z \) can be identified with the elasticity of the individual job finding probability with regard to the search effort \( s \).

Let \( u, n, n_{UI}, n_W, n_R, \) and \( \phi(a, l^+, \epsilon, j) \) denote the unemployment rate, the number of employed workers, unemployed workers receiving unemployment insurance, unemployed workers receiving welfare payments, retired agents, and the measure of the \( j \)-period old people with wealth \( a \), previous labor supply \( l^+ \), and employment status \( \epsilon \), respectively, implying:

\[
u = \frac{n_{UI} + n_W}{n + n_{UI} + n_W}
\]

\[
n = \sum_j \int_a \int_{l^+} \phi(a, l^+ + 1, j) dl^+ da
\]

\[
n_{UI} = \sum_j \int_a \int_{l^+} \phi(a, l^+, 2, j) dl^+ da
\]

\[
n_W = \sum_j \int_a \phi(a, 0, 3, j) da
\]

\[
\frac{T_R}{T + T_R} = \sum_j \int_a \int_{l^+} \phi(a, l^+, 4, j) dl^+ da \equiv n_R,
\]

\[
1 = n + n_{UI} + n_W + n_R.
\]

\(^6\)By this choice of its functional form, the employment probability is bounded between zero and one.
Effective labor $N$ in the economy equals the number of total working hours:

$$N = \sum_j \int_a \int_{l^+} \phi(a, l^+, 1, j) l(a, j) dl^+ da,$$  \hspace{1cm} (10)

where $l(a, j)$ is the labor supply of the $j$-year-old employed agent with wealth $a$. The average labor supply $\bar{I}$ is equal to effective labor $N$ divided by the number of employed workers $n$, $\bar{I} = N/n.$

2.2 Government

The government uses the revenues from taxing labor in order to finance its expenditures on social security and lump-sum transfers $tr$:

$$\tau w N = \sum_j \int_a \int_{l^+} wU(l^+) \phi(a, l^+, 2, j) dl^+ da + w_W n_W + w_R n_R + tr.$$  \hspace{1cm} (11)

The government policy is characterized by the set $\Omega = \{w_{\min}, \theta, \vartheta_W, \vartheta_R, \tau, tr\}$, where $\vartheta_x = \frac{x}{(1+r)x}$ denotes the replacement ratio of $x \in \{w_W, w_R\}$.

2.3 Firms

Firms are of measure one. They hire effective labor $N$ and capital $k$ in order to produce output $Y = A_0 k^\alpha N^{1-\alpha}$. Let $V(N_t, k_t)$ denote the value function of the firm in period $t$:

$$V(N_t, k_t) = \max_{i_t,h_t} \left[ A_0 k_t^\alpha N_t^{1-\alpha} - w_t N_t - \kappa h_t - i_t + \frac{1}{1+r} V(N_{t+1}, k_{t+1}) \right],$$  \hspace{1cm} (12)

where $i_t$ and $h_t$ denote investment and the number of vacancies in period $t$, respectively. The opening of vacancies requires an investment of $\kappa$ units of output.
Every period, workers separate from employment with the exogenous probability $\delta$:

$$ n_{t+1} = q_t h_t + (1 - \delta)n_t. \quad (13) $$

The firm takes the hiring probability $q_t$ as exogenous. The capital stock accumulates according to

$$ k_{t+1} = i_t + (1 - \delta_k)k_t, \quad (14) $$

where the depreciation rate of capital is given by $\delta_k$. In steady state, employment $n$, average labor supply $\bar{l}$, capital $k$, investment $i$, and hiring probability $q$ are all constant, and the Euler equations of the firm are described by:

$$ r + \delta_k = A_0 \alpha k^{\alpha - 1} N^{1 - \alpha} $$

$$ r + \delta = \frac{q}{k} \left( A_0 (1 - \alpha) k^\alpha N^{-\alpha} \bar{l} - w \bar{l} \right) $$

$$ q h = \delta n $$

$$ i = \delta k. $$

Furthermore, firms pay dividends $d = A_0 k^{\alpha} N^{1 - \alpha} - w N - i - \kappa h$ to the asset owners.

### 2.4 Matching and Wage Determination

Labor markets are subject to frictions and are characterized by two-sided search. Unemployed agents search with intensity $s$, and firms post vacancies $h$. The steady state matching coefficients $\pi$ and $q$ are given by:

$$ \pi = \frac{h}{h^{1/\rho} + (n_{UI} + n_W)^{1/\rho}} \quad (19) $$

$$ q = \frac{1}{h} \sum_{e \in \{2,3\}} \sum_j \int_0^l \int_l^{l+} \phi(a, l^+, \epsilon, j) \left(1 - e^{-\pi \tilde{s}}\right) \, dl^+ \, da. \quad (20) $$

---

7 By this choice, which follows den Haan et al. (1997) and Costain (1997), the equilibrium number of aggregate matches is approximately equal to $qh \approx \sum_{i,j} \int_a^l \int_{l^+} \phi(a, l^+, \epsilon, j) \pi s^\epsilon \, dl^+ \, da \approx \frac{\mu^\psi (n_{UI} + n_W)^{h}}{(h^{1/\rho} + (n_{UI} + n_W)^{1/\rho})^{\psi}}$, which depends positively on average search effort $\tilde{s}$. The aggregate matching function is increasing in its arguments $h, n_{UI} + n_W$, and $\tilde{s}$, and has constant returns to scale in its first two arguments.
Wages result from collective bargaining at the firm level. The insiders are represented by an employed worker whose consumption is equal to the average consumption of all employed workers. Both the firm and the employed workers receive a rent from a successful match. The wage which results from a bargaining process depends on both the fall-back position of the firms and the fall-back position of the representative insider. The fall-back of the firm is given by zero production, while the fall-back of the representative insider is the state of insured unemployment. Under these simplifying assumptions, the wage is an additively separable function of the marginal product of effective labor, \( A_0(1 - \alpha)k^\alpha N^{-\alpha} \), average unemployment insurance \( \overline{u}_U \), the disutility from average working time, \( \overline{t} / \gamma \), and the disutility from searching, \( s_{UI}^2 / \gamma \), with \( s_{UI} \) denoting the average search effort of the unemployed workers eligible for unemployment insurance:

\[
    w = \psi_1 A_0(1 - \alpha)k^\alpha N^{-\alpha} + \frac{\overline{t} \gamma}{\gamma} + \psi_3 \overline{u}_U - \psi_4 \frac{s_{UI}^2}{\gamma}.
\]

(21)

The parameters \( \psi_i > 0, i = 1, \ldots, 4 \), are functions of the endogenous variables \( \pi, \tau, r, c, \) and \( s_{UI} \) and are derived in the appendix.

### 2.5 Stationary Equilibrium

The concept of equilibrium applied in this paper uses a recursive representation of the consumer’s problem following Stokey et al. (1989). Let \( W(a, l^+, \epsilon, j) \) be the value of the objective function of a \( j \)-period old agent with beginning-of-period asset holdings \( a \), previous labor supply \( l^+ \), and employment status \( \epsilon \). \( W(a, l^+, \epsilon, j) \) is defined as the solution to the dynamic program:

\[
    W(a, l^+, \epsilon, j) = \max_{c, a', l, s} \left[ \frac{c^{1-\sigma} - 1}{1 - \sigma} - \frac{(l + s)^\gamma}{\gamma} + \beta E \left\{ W(a', l^{+'}, \epsilon', j + 1) \right\} \right],
\]

subject to the budget constraint (2). \( E \), again, denotes the expectation operator conditional on information at age \( j \), while \( a', l^{+'} \), and \( \epsilon' \) are the next-period values of \( a, l^+ \), and \( \epsilon \).
respectively. In the next period,

\[
l_{j+1}^+ = \begin{cases} 
  l_j & \epsilon_j = 1 \\
  l_j^+ & \epsilon_j = 2, 4 \\
  0 & \epsilon_j = 3
\end{cases}
\]

(23)

**Definition**

A Stationary Equilibrium for a given set of government policy parameters

\[ \Omega = \{w_{\min}, \theta, \theta_W, \theta_R, tr, \tau\} \]

is a collection of value functions \( W(a, l^+, \epsilon, j) \) of the households and \( V(N, k) \) of the firms, individual policy rules \( c(a, l^+, \epsilon, j), s(a, l^+, \epsilon, j), l(a, j) \), and \( a'(a, l^+, \epsilon, j) \), age-dependent, time-invariant measures of agent types \( \phi(a, l^+, \epsilon, j) \) for each age \( j = 1, 2, \ldots, T + T^R \), relative prices of labor and capital \( \{w, r\} \), such that:

1. Given relative prices \( \{w, r\} \) and the government policy \( \Omega \), the individual policy rules

   \( c(\cdot), s(\cdot), l(\cdot), \) and \( a'(\cdot) \) solve the consumer’s dynamic program (22) and firms maximize

   profits (12) with respect to investment \( i \) and vacancies \( h \).

2. The goods market clears:

\[
A_0k^\alpha N^{1-\alpha} = \sum_{\epsilon, j} \int_{l^+} c(a, l^+, \epsilon, j) \phi(a, l^+, \epsilon, j) dl^+ da + i + \kappa h.
\]

(24)

3. Households hold equity of the firms. The interest earnings by the households on the

   assets are equal to the dividend payments \( d \) by the firms:

\[
r \sum_{\epsilon, j} \int_{l^+} \phi(a, l^+, \epsilon, j) a dl^+ da = d = A_0k^\alpha N^{1-\alpha} - wn - \kappa h - i.
\]

(25)

4. In each period, \( \int_a l^+ \phi(a, 1, T) dl^+ da \) agents retire from work and the fraction \( 1 - \delta \)

   of these jobs is inherited by the newborn generation, implying:

\[
p_0 = \frac{(1 - \delta) \int_a l^+ \phi(a, l^+, 1, T) dl^+ da}{1/(T + T^R)}.
\]

(26)
5. Wages $w$ result from decentralized bargains according to equation (21).

6. The number of total matches formed in the labor market is equal to the number of job outflows:

$$q_h = \sum_{i,j} \int a \int l^+ \phi(a, l^+, \epsilon, j) \left(1 - e^{-\pi_i(a, l^+, x, j)^\gamma}\right) dl^+ da = \delta n. \quad (27)$$

7. Finally, the government budget (11) is balanced.

## 2.6 Calibration

The steady state distribution of wealth, search effort, and employment and the effects of a change in the unemployment compensation system on employment and welfare cannot be studied analytically but only numerically. For this reason, the model is calibrated in order to match characteristics of the German economy after unification. The time series data refer to the period 1991-97. Time periods correspond to years.

### Households

The coefficient of relative risk aversion $\sigma$ is set equal to 2.0.\footnote{All our qualitative results also hold for the case $\sigma \in \{1, 4\}$.} We choose the value 0.995 for household’s discount factor $\beta$ implying an annual real interest rate equal to 2.85%. $\gamma$ is calibrated in order to imply a labor supply elasticity of $\epsilon = 1/(1-\gamma) = 0.4$ as suggested by Shi/Wen (1999). The calibration of the model’s parameters is summarized in table 1.

### Government

The government provides unemployment insurance, welfare payments, and public pensions. In accordance with Heer (2003), the replacement ratios of welfare payments and public
Table 1: Calibration of parameter values for the German economy

<table>
<thead>
<tr>
<th>Description</th>
<th>Function</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>utility function</td>
<td>$U_t = \frac{e^{1-\sigma} - 1}{1-\sigma} - \frac{(l+s)^\gamma}{\gamma}$</td>
<td>$\sigma = 2.0, \gamma = 3.5$</td>
</tr>
<tr>
<td>discount factor</td>
<td>$\beta$</td>
<td>$\beta = 0.995$</td>
</tr>
<tr>
<td>production function</td>
<td>$Y = A_0 k^\alpha n^{1-\alpha}$</td>
<td>$\alpha = 0.35, A_0 = 1$</td>
</tr>
<tr>
<td>depreciation</td>
<td>$\delta_k$</td>
<td>$\delta_k = 0.04$</td>
</tr>
<tr>
<td>job separation</td>
<td>$\delta$</td>
<td>$\delta = 0.10$</td>
</tr>
<tr>
<td>matching function</td>
<td>$\pi = \frac{\mu (h^{1/p} + (m_U + m_W)^{1/p})}{1 - e^{-x^p}} z = 0.4$</td>
<td>$\rho = 0.78, \mu = 5.9$</td>
</tr>
<tr>
<td>bargaining power</td>
<td>$\lambda$</td>
<td>$\lambda = 0.5$</td>
</tr>
<tr>
<td>unemployment insurance</td>
<td>$w_{\text{min}}$, $\theta$</td>
<td>$\theta_{UI} \equiv \frac{w_{UI}}{(1-\tau)w_I} = 50%$, $w_{\text{min}} = w_W$, $\theta = 0.20$</td>
</tr>
<tr>
<td>welfare payments</td>
<td>$w_W$</td>
<td>$\theta_W \equiv w_W/(1-\tau)w_I = 30%$</td>
</tr>
<tr>
<td>pension payments</td>
<td>$w_R$</td>
<td>$\theta_R \equiv w_R/(1-\tau)w_I = 50%$</td>
</tr>
<tr>
<td>transfers</td>
<td>$tr$</td>
<td>$tr = 0$</td>
</tr>
</tbody>
</table>

Pensions are set equal to 30% and 50%, respectively. In our benchmark case, the minimum unemployment insurance payments $w_{\text{min}}$ are set equal to the welfare payments $w_W$. The progressivity index $\theta = 0.2$ is chosen in order to imply a replacement ratio of unemployment insurance equal to 50%. Transfers $tr$ are set equal to zero in the benchmark case. The income tax rate $\tau$ is calculated endogenously from the government budget (11) and amounts to 24.95%.

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9We also computed the equilibrium values for different values of the replacement ratio of welfare payments ($\theta_W = 20\%$), the replacement ratio of pensions ($\theta_R = 40\%$), and transfers (equal to 1% and 5% of GDP). Our qualitative results are the same as in the benchmark case.
Production

Andolfatto (1996) argues that vacancy costs $\kappa h$ should not exceed one percent of total production $Y$. In our calibration, $\kappa$ is set equal to 0.10 implying vacancy costs at the amount of 0.59% of total production.\footnote{Applying higher values of $\kappa \in \{0.15, 0.20\}$ resulted in no change of our qualitative results.} The remaining production parameters are taken from Heer (2003). In particular, the production elasticity of capital is set equal to $\alpha = 0.35$, $A_0$ is normalized to one, and the annual depreciation rate in Germany amounts to $\delta_k = 0.04$.

Labor Market and Disutility from Labor and Searching

The parameters of the matching functions $z = 0.4$ is taken from Costain (1997), while $\rho = 0.78$ is taken from den Haan et al. (1997). The bargaining power of workers $\lambda$ is set equal to 0.5 implying a labor income share equal to 64.2%. The separation probability $\delta = 10\%$ is taken from Heer (2003). Finally, the matching parameter $\mu = 5.9\%$ is chosen in order to match the average German unemployment rate of 10.0% during 1991-97.

3 Results

In this section, we study the effects of a change in the indexation of unemployment insurance benefits to previous earnings, keeping total government expenditures on unemployment insurance payments constant. The benchmark is given by a lump-sum component of unemployment insurance equal to $w_{min} = w_W = 0.363$ and a proportionality parameter $\theta = 20\%$ of the unemployment insurance component relative to net previous labor earnings. The benchmark is illustrated in the third row of table 2.

In our benchmark equilibrium, the real interest rate $r$ is equal to 2.85% corresponding to a capital-output ratio equal to $k/Y = 5.13$. In the years 1991-97, the ratio of the capital stock to annual GDP was equal to 5.0 in Germany. Consumption is an increasing function
of age as the subjective discount rate of the households $1/\beta - 1 = 0.5\%$ is lower than the real interest rate. The aggregate capital stock amounts to $k = 7.744$.\textsuperscript{11} As typically found in life-cycle models, the age-wealth profile is hump-shaped (not illustrated). During the working life of agents, average wealth of each generation increases before it declines after retirement. The distribution of wealth, however, is more equal than the one observed empirically; in our model, the Gini coefficient is equal to 0.28 and falls short of values close to 0.59-0.89 as reported by Bomsdorf (1989).\textsuperscript{12} The main reason why our model underestimates the inequality of wealth dispersion is the negligence of i) heterogenous labor productivity, ii) self-employment, and iii) bequests.\textsuperscript{13}

| $\theta$ | $w_{\text{min}}$ | $u$ | $w$ | $\tau$ | $Y$ | $k$ | $\bar{t}$ | $s_{\text{UI}}$ | Gini | $\Delta_c$ |
|---|---|---|---|---|---|---|---|---|---|
| 0% | 0.606 | 10.05% | 1.545 | 24.97% | 1.501 | 7.677 | 1.039 | 0.312 | 0.281 | -0.096% |
| 10% | 0.486 | 10.05% | 1.547 | 24.95% | 1.505 | 7.706 | 1.041 | 0.312 | 0.280 | -0.054% |
| 20% | 0.363 | 10.04% | 1.549 | 24.95% | 1.508 | 7.744 | 1.042 | 0.315 | 0.281 | 0 |
| 30% | 0.240 | 10.04% | 1.552 | 24.95% | 1.512 | 7.793 | 1.043 | 0.318 | 0.280 | 0.148% |
| 40% | 0.116 | 10.04% | 1.555 | 24.95% | 1.517 | 7.832 | 1.045 | 0.320 | 0.280 | 0.186% |
| 48.7% | 0 | 10.03% | 1.558 | 24.94% | 1.520 | 7.853 | 1.047 | 0.320 | 0.281 | 0.302% |

The household policy functions behave as expected. Consumption is an increasing function of income and wealth, while both the labor supply and the search effort decline with increasing wealth. Furthermore, average labor supply (as presented in figure 1) and search effort (not displayed) decline with age as i) older agents are characterized by higher wealth and ii) the life-time utility from working or searching declines with age. If the employed

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\textsuperscript{11} In equilibrium, aggregate savings $A$ and the capital stock $k$ can be shown to be related by $k = A - n/q$.\textsuperscript{12} Bomsdorf (1989) analyzes the Gini coefficient of savings, securities, and real estate in Germany in 1983 with Gini coefficients being equal to 0.59, 0.89, and 0.74, respectively.\textsuperscript{13} Quadrini and Ríos-Rull (1997) review recent studies that explain the wealth distribution in general equilibrium models with the help of idiosyncratic shocks to labor earnings, business ownership, and changes in health and marital status, among others. Heer (2001) studies the effect of bequests on the distribution of wealth.
worker loses his job, the expected discounted working-life-time income from unemployment insurance payments for given labor supply \( l \) decreases with age. Similarly, as the potential employment period is getting shorter with increasing age, the expected discounted working-life-time income of an unemployed agent is also declining. In the final period of his working life at age \( T \), unemployed agents do not search any more as they will be unable to work in the next period.

**Figure 1: Labor supply-age profiles for different values of \( \theta \)**

In our benchmark case, the unemployment rate is equal to 10.0%. The average labor supply \( \bar{l} \) of the employed workers is equal to 1.042 implying aggregate effective employment \( N = 0.625 \) and output \( Y = 1.508 \). An increase of the proportionality factor \( \theta \) results in an increase of both the average labor supply \( \bar{l} \) and output \( Y \). However, the quantitative effect of a higher indexation \( \theta \) on labor supply is negligible (compare also figure 1). Increasing \( \theta \) from 20\% to 40\% (with a corresponding reduction of the lump-sum component \( u_{\text{min}} \) from 0.363 to 0.116) only results in an increase of the average labor supply from 1.042 to 1.045. Strikingly, however, we find an increase of output by 0.6\%, from 1.508 to 1.517. This effect is caused by an increase of precautionary savings that is also reflected in a
rise of the capital stock from 7.744 to 7.832. Employed agents can insure against the bad
luck of unemployment by either i) increasing their labor supply and, hence, the amount
of unemployment insurance payments entitled to, or ii) increasing their savings. In our
economy, the second form of insurance predominates.

Notice that wages \( w \) slightly increase with a higher earnings-related benefit scheme. The
equilibrium wage at \( \theta = 40\% \) is 0.6% above the one at \( \theta = 0\% \). There are two effects that
affect the wage bargain (21). First, as shown by Goerke/Madsen (1999), the increase of
\( \theta \) for constant unemployment benefits \( w_{UI} \) results in a decline in wages. The intuition of
this result is straightforward: a decrease of \( w \) decreases both the worker’s income during
employment and his fall-back position so that the worker’s relative utility loss is smaller
than the relative firm’s gain from an increase in profits.\(^{14}\) Second, the marginal product of
labor increases as the capital stock increases. In our calibration, the second effect dominates
and wages slightly increase with higher indexation of UI payments to past contributions.
Notice further that the search effort \( s_{UI} \) is not affected by \( \theta \), as \( s_{UI} \) only depends on the
individuals’ total amount of unemployment compensation, but not on its composition. On
average, however, unemployment insurance payments remain almost constant for different
values of \( \theta \) as we keep government expenditures on unemployment compensation fixed in all
our computations. As a consequence, employment does not vary on the extensive margin,
with an almost constant unemployment rate equal to \( u = 10.0\% \).

As we keep total government expenditures on unemployment compensation constant, the
different policy schemes \( \{ \theta, w_{\text{min}} \} \) presented in table 2 also redistribute the same amount
of income from employed to unemployed workers. Accordingly, the distribution of wealth
is almost unaffected and the Gini coefficient remains unchanged. Similarly, the income
tax \( \tau \) that is necessary in order to finance government expenditures on unemployment
compensation and pensions is also fairly constant and is approximately equal to 25.0%.

In order to compare the welfare effects of alternative unemployment compensation schemes,
\(^{14}\)The effect is similar to the observation of Lockwood/Manning (1993) and Koskela/Vilkmanen (1996)
that a more progressive income tax system results in lower wages from bargaining.
we need a measure of average utility. For government policy \( \Omega \), we measure welfare by the expected discounted life-time utility of the newborn generation which is simply equal to 
\[
W(\Omega) = p_0 W(0, 0, 1, 1) + (1 - p_0) W(0, 0, 3, 1).
\]
In order to quantify the welfare benefits, we take the benchmark equilibrium as presented in table 1 as our reference economy. The change in welfare \( \Delta_w \) is computed as the compensation in consumption (relative to the reference economy) required in order to make the average newborn indifferent between the reference economy and an economy under an alternative unemployment compensation arrangement. As presented in table 2, there is a small, but noticeable effect of the indexation scheme on welfare. Welfare increases with higher indexation and for the case with no lump-sum unemployment insurance, \( w_{\text{min}} = 0 \) and \( \theta = 48.6\% \), steady-state welfare gains compared to the benchmark case amount to 0.302\% of total consumption.

We, however, need to be careful when we interpret these welfare results. In this study, we only consider the steady state and do not compute the transition dynamics. In our model, steady-state life-time utility increases with higher indexation \( \theta \). The main difference between an economy with lump-sum unemployment benefits and an economy with unemployment benefits proportional to previous earnings is the amount of precautionary savings as reflected by a higher value of the aggregate capital stock \( k \). With higher savings in the latter case, welfare increases because aggregate savings are sup-optimal in the presence of lump-sum pensions. Changing the current unemployment compensation scheme to one with complete proportional unemployment benefits, however, results in transition dynamics that are characterized by higher savings than in the new steady state and, hence, imply a transitory decrease of average utility. Consequently, also considering the transition dynamics following a change in policy, welfare effects will be less than those presented in table 2.
4 Conclusion

We analyze the effects of a change in the unemployment insurance scheme from one that pays lump-sum benefits relative to one that is characterized by benefit payments proportional to past contributions. We find that in the latter case, employment on the intensive margin is higher, while employment on the extensive margin is not affected. The increase in average labor supply, however, is negligible. In addition, employed workers are found to increase precautionary savings in the latter case resulting in a higher equilibrium output. The welfare effects from an unemployment scheme with unemployment insurance benefits that are proportional to earnings, however, are shown to be modest.

In our analysis, we assume that the household consists of one worker. We neglect any effects resulting from the composition of households. One possible important effect of the indexation of unemployment benefits to previous contributions might result from the consideration of two-person households. Households may be composed of an employed worker and the employed/unemployed spouse. Indexation of unemployment benefits to previous earnings might affect the decision of the spouse to work, e.g., part-time. The incentives to work part-time are increased if unemployment insurance payments are provided lump-sum irrespective of previous contributions. Such a scheme will result in higher total employment, even though problems associated with the moral hazard of the job retention effort will be accentuated as well.

5 Appendix

5.1 Deriving the Wage Equation

The derivation of the wage equation is analogous to the one in Heer (2003). In order to derive the wage equation (21), we need the simplifying assumption that wages are bargained between the firms and the representative employed worker whose consumption
is equal to the average consumption of all employed workers and whose fall-back is the value of the representative insured unemployed worker. Given this assumption, the value of employment, \( W^e \), and the value of insured unemployment, \( W^{UI} \), are given by the following Bellman equations:

\[
W^e = (1 - \tau) l w M U^e - \frac{\bar{y}}{\gamma} + \beta \left( (1 - \delta) W^e + \delta W^{UI} \right)
\]

\[
W^{UI} = w^{UI} MU^{UI} - \frac{\bar{y}}{\gamma} + \beta \left( (1 - e^{-\pi^{UI} \delta}) W^e + \pi^{UI} W^{UI} \right)
\]

where \( MU^e \) and \( MU^{UI} \) denote the marginal utility of the representative employed worker and the representative unemployed worker receiving \( w^{UI} \).

In the stationary equilibrium, the value of a filled vacancy to the firm is given by:

\[
V^f = A_0(1 - \alpha) \left( n \bar{l} \right)^{-\alpha} k^{\alpha} l - w \bar{l} + \frac{1}{1 + r} \left[ (1 - \delta) V^f + \delta V^v \right],
\]

and the value of an unfilled vacancy is equal to zero:

\[
V^v = 0.
\]

Wages are bargained in every period according to the Nash bargaining solution where the fall-back position of the insiders is the state of short-term insured unemployment, \( W^{UI} \).

The wage rate satisfies:

\[
w = \arg \max_w \left( W^e - W^{UI} \right)^{1-\lambda} \left( V^f - V^v \right)^{\lambda},
\]

\(^{15}\)In our numerical computations, the marginal utility of the representative employed worker is calculated using the average consumption of employed workers \( \bar{e}^e, MU^e = (\bar{e}^e)^{-\gamma} \). Similarly, \( MU^{UI} \) is computed using the average consumption levels of the workers who receive unemployment insurance.
where $\lambda$ denotes the bargaining power of the workers. From (28), (29), (30), and (31), the first-order condition of program (32) reads as:

$$
(1 - \lambda) \frac{W^e - W^{UI}}{(1 - \tau)MU^e} = \lambda \left( V^F - V^v \right),
$$

(33)

After some tedious algebra, the wage equation (21) can be derived with the help of the equations (28)-(33). The parameters $\psi_i, i = 1, \ldots, 4$ are given by:

$$
\psi_1 = \frac{\lambda}{1} \frac{1 + r}{1 - \lambda r + \delta},
$$

$$
\psi_2 = \frac{\zeta_0 \zeta_1 \zeta_3}{(1 - \tau)MU^e} \frac{1}{MU^{UI}},
$$

$$
\psi_3 = \frac{\zeta_2 \zeta_3}{(1 - \tau)MU^e},
$$

$$
\psi_4 = \frac{\zeta_2 \zeta_3}{(1 - \tau)MU^e}.
$$

where

$$
(\zeta_0)^{-1} = 1 - \beta (1 - \delta) - \beta^2 \delta \frac{1 - e^{-\pi \phi t}}{1 - \beta e^{-\pi \phi t}},
$$

$$
\zeta_1 = 1 - \beta \frac{1 - e^{-\pi \phi t}}{1 - \beta e^{-\pi \phi t}},
$$

$$
\zeta_2 = \frac{1 - \zeta_0 \zeta_1 \beta \delta}{1 - \beta e^{-\pi \phi t}},
$$

$$
(\zeta_3)^{-1} = \zeta_0 \zeta_1 + \frac{\lambda}{1 - \lambda r + \delta}.
$$

5.2 The Solution Algorithm

The model has no analytical solution. Algorithms to solve heterogenous-agent models with an endogenous distribution have only recently been introduced in the economic literature. Notable studies in this area are Aiyagari (1994), Costain (1997), den Haan (1996), Huggett (1993), and İmrohoroğlu et al. (1995). Like most of these studies, we will only focus on the steady state of the model. Our algorithm is described by the following steps:
1. Choose the policy parameters $w_{\text{min}}, \theta, tr, \theta_{UI}, \theta_W,$ and $\theta_R$.

2. Make initial guesses of $r, \pi,$ and $w$.

3. Compute the household's decision function by backwards induction.

4. Compute the steady-state distribution of assets, employment, consumption, labor supply, and search effort.

5. Compute $n, n_{UI}, n_{UA}, n_W, \bar{l}$, average consumption of the employed workers, average consumption and search effort of the unemployed workers, and the average asset holdings of all households.

6. Compute the values $\tau, k, q, i, w, r,$ and $\pi$ that solves the firm's Euler equations, the wage equation, the government budget, and the aggregate consistency conditions.

7. Update $r, \pi, w,$ and $w_{\text{min}},$ and return to step 3 if necessary.

In step 3, a simple finite-time dynamic programming problem is solved with value function iteration. We choose a grid over the asset space $\{a, \epsilon, l^+\}$ with $a \in [0, a^{\max}], \epsilon \in \{1, 2, 3, 4\},$ and $l^+ \in [0, l^{\max}]$ such that the upper limits $a^{\max} = 20.0$ and $l^{\max} = 3.0$ never bind. The grid is chosen to consist of $n = 1000$ equidistant points.\footnote{Increasing the number of gridpoints did not result in any change of equilibrium values.} In step 4, the steady-state distribution is computed by forward iteration starting with the 21-year old (corresponding to the 1-period old generation in the model) who has no wealth and given employment probability $p_0$.\footnote{A more detailed description of the numerical computation of the stationary distribution can be found in Huggett (1993).} The algorithm stops as soon as two successive values of $w, r,$ and $\pi$ diverge by less than 0.1\%, respectively.
References


