The Burden of Unanticpated Inflation: Analysis of an Overlapping Generations Model with Progressive Income Taxation and Staggered Prices*

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Abstract

Inflation is often associated with a loss for the poor in the medium and long run. The cyclical effects are basically unknown. We study this question in a dynamic optimizing sticky price model. Agents are heterogeneous with regard to their age and their productivity. We emphasize three channels through which unanticipated inflation affects the income and wealth distribution: 1) factor prices, 2) the 'bracket creep', and 3) sticky pensions. In our model, unanticipated inflation decreases the inequality of factor income, while the redistributive effect of inflation on total income depends on whether the government is spending the additional revenues on transfers or public consumption.

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1 Introduction

The redistributive effects of inflation have been analyzed in numerous studies of the US economy. Galli and van der Hoeven (2001) provide a survey of the empirical literature. In the majority of the studies, inflation is found to increase income inequality. Easterly and Fischer (2001), Bulir (2001), and Romer and Romer (1998) also point out in their empirical analysis that inflation hurts the very poor in the high-inflation countries. However, the evidence is inconclusive for countries with low inflation rates. Doepke and Schneider (2005, 2006) document the effect of inflation on the holdings of nominal assets for US households depending on age and groups. They show that redistribution takes the form of 'ends-against-the middle' meaning that the middle class gains at the expense of the rich and poor.

To the best of our knowledge, all these empirical studies analyze the effects of inflation at low frequencies. While most studies use the annual data set of Deininger and Squire (1996), Doepke and Schneider look at a hypothetical 10-year return of a high-inflation period that redistributes income from the lender to the borrower among the current US households. For this reason, they use data from the 1989 and 2001 Survey of Consumer Finances (SCF). However, we do not know of any empirical or dynamic general equilibrium (DGE) study that considers inflation and income inequality at business cycle frequencies with quarterly or monthly data. Consequently, the cyclical redistributive effects of inflation are basically unknown.

This paper presents a DGE model that is able to study the impact of inflation on the distribution of both income and wealth over the business cycle. Therefore, it provides a first step to the analysis of the distributional effects of monetary policy. Our model is an extension of a standard Neo-Keynesian model. In order to model the short-run effects of monetary policy, we assume that prices are sticky and adjust as in Calvo (1983). Following an unexpected rise of the money growth rate, we observe unexpected inflation. Prices and markups adjust endogenously in our economy. In light of the contribution of Doepke and Schneider (2006), we also account for the fact that the effect of inflation on individual income and wealth depends on age and idiosyncratic productivity. Therefore, we consider an Overlapping Generations model that is able to replicate the empirical distribution of income and wealth closely. We emphasize three channels of inflation on income inequality: 1) factor prices, 2) 'bracket creep', and 3) sticky pensions.

1. Following a monetary expansion, demand goes up and factor prices increase. As a consequence, we also observe unanticipated inflation. The increase in wages results in higher labor income among all workers, but especially among the younger and less productive workers as the income tax is progressive.

2. In addition, we model the so-called 'bracket creep' effect. The tax brackets are only adjusted to actual inflation with a lag. Therefore, if unanticipated inflation increases, the income-rich agents face higher marginal and average tax rates and inflation redistributes income to the poor.

3. Pensions are also indexed to actual inflation with a lag. Therefore, unanticipated inflation redistributes income from the old to the young.

In general, we find that inflation increases the factor income of the poor workers relative to that of the rich workers.[1] The redistributive effect of inflation on total after-tax income including transfers and pensions, however, depends crucially on the modeling of the government sector. If higher tax revenues and savings from lower pension payments are used in order to finance more government consumption rather than lump-sum transfers, unanticipated inflation is found to increase the inequality of the total income distribution.

Our analysis is most closely related to the general equilibrium model of Cysne, Maldonado, and Monteiro (2005) who consider a shopping-time economy. In their model, consumers have different access to a shopping time technology. In particular, poor agents cannot use interest-bearing nominal bonds in order to economize on transaction costs. Accordingly, rich agents are able to pay a smaller inflation tax than poor agents. While Cysne et al. (2005), however, do not consider sticky prices and only analyze the steady-state behavior of their economy, we calibrate our dynamic general equilibrium model and consider the cyclical responses of income inequality to a monetary shock. In addition, we also provide for a realistic modeling of the wealth distribution which, of course, is important when we study the redistributive channel of inflation that works through its effect on the return of assets.

Our paper is also related to the recent work by Erosa and Ventura (2002) and Heer and Süssmuth (2007) on the effects of inflation on wealth inequality. Both studies consider only changes in the long-run inflation rate that are fully anticipated. They

[1] When we talk about the poor, we refer to the income-poor if not mentioned otherwise.

find that a rise in the anticipated long-run inflation rate results in an increase of the wealth inequality. Erosa and Ventura (2002) emphasize the effect of inflation on the composition of the consumption good bundle. Higher inflation results in an increase of the consumption of the credit good at the expense of the consumption of the cash good, and richer agents have lower credit costs. Heer and Süssmuth (2007) model the observation that not all agents have access to the stock market and, therefore, poorer agents are less likely to hold assets whose real return is not reduced through higher inflation. In addition, they consider the so-called 'Feldstein channel' according to which higher inflation reduces real after-tax interest income. All these studies, however, refrain from modeling the effects of unanticipated inflation.

The paper is organized as follows. In section 2, we describe the OLG model. The model is calibrated in section 3, where we also describe the algorithm for our computation in brief. Section 4 presents the results upon the effects of a monetary shock on the distribution of income and wealth. Section 5 concludes. A more detailed description of the stochastic OLG and the solution method is delegated to the Appendix.

2 The model

The model is based on the stochastic Overlapping Generations (OLG) model with elastic labor supply and aggregate productivity risk, augmented by a government sector and the monetary authority. The model is an extension of Ríos-Rull (1996).

Four different sectors are depicted: households, firms, the government, and the monetary authority. Households maximize discounted life-time utility with regard to their intertemporal consumption, capital and money demand, and labor supply. Firms in the production sector are competitive, while firms in the retail sector are monopolistically competitive and set prices in a staggered way as in Calvo (1983). The intermediate good firms are using labor and capital as input into production. The government taxes income progressively and spends the revenues on government pensions and transfers. Monetary policy is stochastic.

2.1 Households

Households live $T + T^R = 240$ periods (corresponding to 60 years). Each generation is of measure $1/(T+T^R)$. The first T=160 (=40 years) periods, they are working, the last

$T^R = 80$ periods (=20 years), they are retired and receive pensions. Agents are also heterogeneous with regard to their productivity level $e(s,j)$. The productivity $e(s,j)$ depends on the age s and the productivity type $j \in \{1,\ldots,ne\}$ at birth. Individual productivity is non-stochastic, and an individual will not change his productivity type j over his life time. The mass of type-j agents in each generations is constant and denoted by $\mu(j)$. The s-year old household with productivity type j holds real money holdings $M_t^{s,j}/P_t$ and capital $k_t^{s,j}$ in period t. He maximizes expected life-time utility at age 1 in period t with regard to consumption $c_t^{s,j}$, labor supply $n_t^{s,j}$, and next-period money balances $M_{t+1}^{s+1,j}$, and real capital $k_{t+1}^{s+1,j}$:

$$E_t \sum_{s=1}^{T+T^R} \beta^{s-1} u\left(c_{t+s-1}^{s,j}, \frac{M_{t+s-1}^{s,j}}{P_{t+s-1}}, 1-n_{t+s-1}^{s,j}\right), \tag{1}$$

where β is a discount factor and expectations are conditioned on the information set of the household at time t. Instantaneous utility $u\left(c_t, \frac{M_t}{P_t}, 1-n_t\right)$ is assumed to be:

$$u\left(c, \frac{M}{P}, 1-n\right) = \begin{cases} \gamma \ln c + (1-\gamma)\ln\frac{M}{P} + \eta_0 \frac{(1-n)^{1-\eta}}{1-\eta} & \text{if } \sigma = 1 \\ \frac{\left(c^\gamma \left(\frac{M}{P}\right)^{1-\gamma}\right)^{1-\sigma}}{1-\sigma} + \eta_0 \frac{(1-n)^{1-\eta}}{1-\eta} & \text{if } \sigma \neq 1, \end{cases} \tag{2}$$

where $\sigma > 0$ denotes the coefficient of relative risk aversion.[2] The agent is born without capital $k_t^{1,j} = 0$, $j \in \{1,\ldots,ne\}$, but receives an initial cash endowment from the government M_t^{1j} that is fixed in terms of the beginning of period price level P_{t-1} and equal for the different productivity types, i.e., $M_t^{1j}/P_{t-1} =: m^1 > 0$ for all t and j.

The s-year old working agent with productivity type j faces the following nominal budget constraint in period t:

$$\begin{aligned} &P_t\left(k_{t+1}^{s+1,j} - (1-\delta)k_t^{s,j}\right) + \left(M_{t+1}^{s+1,j} - M_t^{s,j}\right) + P_t c_t^{s,j} \\ &= P_t r_t k_t^{s,j} + P_t w_t e(s,j) n_t^{s,j} + P_t tr_t + P_t \Omega_t - P_t \tau_t \left(\frac{P_t y_t^{s,j}}{P_{t-1}\pi}\right), \\ &s = 1,\ldots,T, \quad j = 1\ldots,ne. \end{aligned} \tag{3}$$

The working agent receives income from effective labor $e(s,j)n_t^{s,j}$ and capital $k_t^{s,j}$ as well as government transfers tr_t and profits Ω_t which are spent on consumption $c_t^{s,j}$ and

[2]We follow Castañeda, Díaz-Giminénez, and Ríos-Rull (2003) in our choice of the functional form for the utility from leisure. In particular, this additive functional form implies a relatively low variability of working hours across individuals that is in good accordance with empirical evidence.

next-period capital $k_{t+1}^{s+1,j}$ and money $M_{t+1}^{s+1,j}$. He pays taxes on his nominal income $P_t y_t^{s,j}$:

$$P_t y_t^{s,j} = P_t r_t k_t^{s,j} + P_t w_t e(s,j) n_t^{s,j}.$$

The government adjusts the tax income schedule at the beginning of each period for the average rate of inflation in the economy which is equal to the non-stochastic steady state rate π. Therefore, nominal income is divided by the price level, $P_{t-1}\pi$, and the tax schedule $\tau(.)$ is a time-invariant function of (deflated) income with $\tau' > 0$. Notice that when we have unanticipated inflation, $\pi_t = \frac{P_t}{P_{t-1}} > \pi$, the real tax burden increases as the agent's real income moves into a higher tax bracket, the so-called "bracket creep" effect.

The nominal budget constraint of the retired worker is given by

$$\begin{aligned} & P_t \left(k_{t+1}^{s+1,j} - (1-\delta) k_t^{s,j} \right) + \left(M_{t+1}^{s+1,j} - M_t^{s,j} \right) + P_t c_t^{s,j} \\ & = P_t r_t k_t^{s,j} + Pen_t + P_t tr_t + P_t \Omega_t - P_t \tau_t \left(\frac{P_t y_t^{s,j}}{P_{t-1}\pi} \right), \\ & s = T+1, \ldots, T+T^R, \;\; j = 1, \ldots, ne, \end{aligned} \tag{4}$$

with the capital stock and money balances at the end of the life at age $s = T+T^R$ being equal to zero, $k_t^{T+T^R+1,j} = M_t^{T+T^R+1,j} \equiv 0$ for all productivity types $j \in \{1, \ldots, ne\}$, because the household does not leave bequests. Furthermore, since retirement at age $T+1$ is mandatory, $n_t^{T+1,j} = n_t^{T+2,j} = \ldots = n_t^{T+T^R,j} \equiv 0$. Pen_t are nominal pensions and are distributed lump-sum. They are not taxed. Again, the government adjusts pensions each period for expected inflation according to $Pen_t = pen\ P_{t-1}\pi$, where pen is constant through time. If inflation is higher then expected, $\pi_t > \pi$, the real value of pensions declines.

The real budget constraint of the s-year old household with productivity type j is given by

$$k_{t+1}^{s+1,j} + m_{t+1}^{s+1,j} = \begin{cases} (1+r_t-\delta)k_t^{s,j} + \frac{m_t^{s,j}}{\pi_t} + w_t e(s,j) n_t^{s,j} + tr_t + \Omega_t - \tau_t \left(\frac{y_t^{s,j}\pi_t}{\pi} \right) - c_t^{s,j}, \\ s = 1, \ldots, T, \\ (1+r_t-\delta)k_t^{s,j} + \frac{m_t^{s,j}}{\pi_t} + \frac{pen\ \pi}{\pi_t} + tr_t + \Omega_t - \tau_t \left(\frac{y_t^{s,j}\pi_t}{\pi} \right) - c_t^{s,j}, \\ s = T+1, \ldots, T+T^R, \end{cases} \tag{5}$$

where we define $m_t^{s,j} \equiv \frac{M_t^{s,j}}{P_{t-1}}$.

The necessary conditions for the households with respect to consumption $c_t^{s,j}$, $s = 1, \ldots, T+T^R$, next-period capital $k_{t+1}^{s+1,j}$, and next-period money $m_{t+1}^{s+1,j}$ at age $s = 1, \ldots, T+T^R-1$ in period t are as follows:

$$\lambda_t^{s,j} = u_c\left(c_t^{s,j}, \frac{M_t^{s,j}}{P_t}, 1-n_t^{s,j}\right) = \gamma\left(c_t^{s,j}\right)^{\gamma(1-\sigma)-1}\left(\frac{m_t^{s,j}}{\pi_t}\right)^{(1-\gamma)(1-\sigma)}, \tag{6}$$

$$\lambda_t^{s,j} = \beta E_t\left[\lambda_{t+1}^{s+1,j}\left(1-\delta+r_{t+1}\left(1-\tau'\left(y_{t+1}^{s+1,j}\frac{\pi_{t+1}}{\pi}\right)\frac{\pi_{t+1}}{\pi}\right)\right)\right], \tag{7}$$

$$\begin{aligned}
\lambda_t^{s,j} &= \beta E_t\left[\frac{\lambda_{t+1}^{s+1,j}}{\pi_{t+1}} + \frac{u_{M/P}\left(c_{t+1}^{s+1,j}, \frac{M_{t+1}^{s+1,j}}{P_{t+1}}, 1-n_{t+1}^{s+1,j}\right)}{\pi_{t+1}}\right] \\
&= \beta E_t\left[\frac{\lambda_{t+1}^{s+1,j}}{\pi_{t+1}} + \frac{(1-\gamma)\left(c_{t+1}^{s+1,j}\right)^{\gamma(1-\sigma)}\left(m_{t+1}^{s+1,j}\right)^{(1-\gamma)(1-\sigma)-1}}{\pi_{t+1}}\right].
\end{aligned} \tag{8}$$

The optimal labor supply of the productivity j-type workers at age $s = 1, \ldots, T$ is given by:

$$\lambda_t^{s,j} w_t e(s,j)\left[1-\tau'\left(y_t^{s,j}\frac{\pi_t}{\pi}\right)\frac{\pi_t}{\pi}\right] = u_n\left(c_t, \frac{M_t^{s,j}}{P_t}, 1-n_t^{s,j}\right) = \eta_0\left(1-n_t^{s,j}\right)^{-\eta}. \tag{9}$$

2.2 Production

The description of the production sector is similar to Bernanke, Gertler, and Gilchrist (1999). A continuum of perfectly competitive firms produce the final output using differentiated intermediate goods distributed on [0,1]. These goods are manufactured by monopolistically competitive firms. Firms in the intermediate goods' sector set prices according to Calvo (1983).

Final goods firms. The firms in the final goods sector produce the final good with a constant returns to scale technology using the intermediate goods $Y_t(j)$, $j \in [0,1]$ as an input:

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}. \tag{10}$$

Profit maximization implies the demand functions:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t, \tag{11}$$

with the zero-profit condition

$$P_t = \left(\int_0^1 P_t(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}. \tag{12}$$

Intermediate goods firms. The intermediate good $j \in [0, 1]$ is produced with capital $K_t(j)$ and effective labor $N_t(j)$ according to:

$$Y_t(j) = K_t(j)^\alpha N_t(j)^{1-\alpha}. \tag{13}$$

The firms choose $K_t(j)$ and $N_t(j)$ in order to maximize profits. In a symmetric equilibrium profit maximization of the intermediate goods' producers implies:

$$r_t = g_t \alpha K_t^{\alpha-1} N_t^{1-\alpha}, \tag{14}$$

$$w_t = g_t (1-\alpha) K_t^\alpha N_t^{-\alpha}, \tag{15}$$

where g_t denotes the inverse of the mark-up.

Calvo price setting. Let ϕ denote the fraction of producers that keep their prices unchanged. Profit maximization of symmetric firms leads to a condition that can be expressed as a dynamic equation for the aggregate inflation rate:

$$\hat{\pi}_t = \kappa \hat{g}_t + \beta E_t \left\{ \hat{\pi}_{t+1} \right\}. \tag{16}$$

with $\kappa \equiv (1-\phi)(1-\beta\phi)/\phi > 0$ and $\hat{\pi}_t$ is the percent deviation of the gross inflation rate from its non-stochastic steady state level π.[3]

2.3 Monetary authority

The nominal stock of money held by generations $s = 2$ through $s = T + T^R$, M_t, grows at the exogenous rate θ:

$$\frac{M_{t+1}}{M_t} = \theta_t. \tag{17}$$

The seignorage is transferred lump-sum to the government:

$$Seign_t = M_{t+1} - M_t. \tag{18}$$

[3] A detailed derivation of this relation can be found in Herr and Maußner (2005), Section A.4.

The growth rate θ_t follows the process:

$$\hat{\theta}_t = \rho_\theta \hat{\theta}_{t-1} + \varepsilon_{\theta t}, \tag{19}$$

where $\varepsilon_{\theta t}$ is assumed to be i.i.d., $\varepsilon_{\theta t} \sim N(0, \sigma_\theta^2)$.

2.4 Government

Nominal government expenditures in period t consists of pensions Pen_t, the money endowment of the first generation, government consumption $P_t G_t$, and lump-sum transfers $P_t Tr_t$ to households. Government expenditures are financed by an income tax Tax_t and seignorage:

$$P_t G_t + P_t Tr_t + Pen_t + \sum_{j=1}^{ne} \frac{\mu(j)}{T + T^R} M_t^{1j} = Tax_t + Seign_t. \tag{20}$$

The income tax structure is chosen to match the current income tax structure in the US most closely. Gouveira and Strauss (1994) have characterized the US effective income tax function in the year 1989 with the following function:

$$\tau(y) = a_0 \left(y - \left(y^{-a_1} + a_2 \right)^{-\frac{1}{a_1}} \right), \tag{21}$$

and estimate the parameters with $a_0 = 0.258$, $a_1 = 0.768$ and $a_2 = 0.031$. We use the same functional form for our benchmark tax schedule. The average nominal income in 1989 amounts to approximately \$50,000.[4]

2.5 Equilibrium conditions

1. Aggregate and individual behavior are consistent, i.e. the sum of the individual consumption, effective labor supply, wealth, and money is equal to the aggregate

[4] We follow Castañeda et al. (2004).

level of consumption, effective labor supply, wealth, and money, respectively:

$$C_t = \sum_{j=1}^{ne} \sum_{s=1}^{T+T^R} c_t^{s,j} \frac{\mu(j)}{T+T^R}, \tag{22a}$$

$$N_t = \sum_{j=1}^{ne} \sum_{s=1}^{T} n_t^{s,j} e(s,j) \frac{\mu(j)}{T+T^R}, \tag{22b}$$

$$K_t = \sum_{j=1}^{ne} \sum_{s=1}^{T+T^R} k_t^{s,j} \frac{\mu(j)}{T+T^R}, \tag{22c}$$

$$m_t = \sum_{j=1}^{ne} \sum_{s=1}^{T+T^R} m_t^{s,j} \frac{\mu(j)}{T+T^R}. \tag{22d}$$

2. Households maximize life-time utility (1).

3. Firms maximize profits.

4. The goods market clears:

$$K_t^{\alpha} N_t^{1-\alpha} = C_t + G_t + K_{t+1} + (1-\delta)K_t.$$

5. The government budget (20) balances.

6. Monetary growth (17) is stochastic and seignorage is transferred to the government.

The non-stochastic steady state and the log-linearization of the model at the non-stochastic steady state are described in more detail in the Appendix.

3 Calibration and computation

The OLG model is calibrated with regard to the characteristics of the US postwar economy. We use standard values for the parameters of the model. Periods correspond to quarters. The first $T = 160$ periods, agents are working, the remaining $T^R = 80$ periods they are retired.

3.1 Preferences

β is set equal to 0.99 implying a non-stochastic steady state annual real rate of return equal to $r(1-\tau')-\delta = 4.5\%$ and an annualized capital-output ratio equal to $K/Y = 2.1$. The relative risk aversion coefficient σ is set equal to 2.0. $\eta_0 = 0.26$ is set so that the average labor supply is approximately equal to 1/3, $\bar{n} \approx 1/3$. Furthermore, we choose $\eta = 7.0$ which implies a conservative value of 0.3 for the Frisch labor supply elasticity.[5] γ is chosen so that the (annualized) average velocity of money PY/M is equal to the velocity of M1 during 1960-2002, which is equal to approximately 6.0. This requires $\gamma = 0.981$.

3.2 Government

Pensions are constant and equal for all retired workers in the non-stochastic steady state. We choose a replacement ratio of pensions relative to average net wage earnings ζ equal to 30%, $\zeta = \frac{pen_t}{(1-\bar{\tau})w_t\bar{n}_t}$, where $\bar{n}_t$ and $\bar{\tau}$ are the average labor supply and the income tax rate of the average income in the non-stochastic steady state of the economy, respectively. The calibration of the tax schedule follows Goveira and Strauss (1994). We adjust the parameter a_2 in (21) so that the average (and also the marginal) tax rate on annual average US-income equals the quarterly tax rate on average income in the model. In the benchmark case, government consumption is equal to zero. In the alternative policy regime that we consider, transfers are set equal to a small constant and government consumption always adjust in order to balance the government budget.

3.3 Monetary authority

In accordance with Cooley and Hansen (1995), the quarterly inflation factor is set equal to $\pi = 1.013$. Money growth follows an AR(1)-process. For the postwar US economy, Cooley and Hansen estimate $\rho_\theta = 0.49$ and $\sigma_\theta^2 = 0.0089$. The initial endowment with money equals 21 percent of the average disposable income of the first generation. This

[5] The estimates of the Frisch intertemporal labor supply elasticity $\eta_{n,w}$ implied by microeconometric studies and the implied values of γ vary considerably. MaCurdy (1981) and Altonji (1986) both use PSID data in order to estimate values of 0.23 and 0.28, respectively, while Killingsworth (1983) finds an US labor supply elasticity equal to $\eta_{n,w} = 0.4$. Domeij and Floden (2006), however, argue that these estimates are biased downward due to the omission of borrowing constraints.

is about the amount of money held by the 21 year old US citizens according to the 1994 PSID survey.[6]

3.4 Production

The production elasticity of capital $\alpha = 0.36$ and the quarterly depreciation rate $\delta = 0.019$ are taken from Prescott (1986) and Cooley and Hansen (1995), respectively. The annual quarterly fraction of producers that do not adjust their prices in any quarter is set equal to $\phi = 0.25$. This value is smaller than the value chosen, e.g., in Bernanke et al. (1999), who use $\phi = 0.75$, yet it introduces sufficient nominal rigidity into our model. Following empirical evidence presented by Basu and Fernard (1997), we set the average mark-up at the amount of $1/g = 1.2$ implying a constant elasticity of substitution between any two intermediate goods equal to $\epsilon = 6.0$.

3.5 Individual productivity

The idiosyncratic productivity level is given by $e(s,j) = e^{\bar{y}^s + x^j}$, where $\bar{y}^s$ is the mean log-normal income of the s-year old and x^j is the idiosyncratic component. The mean efficiency index $\bar{y}^s$ of the s-year old is taken from Hansen (1993) and is interpolated to in-between quarters. As a consequence, the model replicates the cross-section age distribution of earnings of the US economy. The age-productivity profile is hump-shaped and earnings peak at age 50 corresponding to the model period 121 (not displayed). With regard to the idiosyncratic component x^j, we follow Huggett (1996) and choose a log-normal distribution of earnings for the 20-year old with a variance equal to $\sigma^2_{y^1} = 0.38$ and mean $\bar{y}^1$. The productivity state x^j is equally spaced and ranges from $-\sigma_{y^1}$ to σ_{y^1}. We discretize the state space by using $ne = 2$ values and normalize e^{x^j} so that $\sum_{j=1}^{ne} \mu(j) e^{x^j} = 1$.[7] For $ne = 2$, we have $\mu(j) = 0.5$ for $j = 1, 2$. Table 1 summarizes our choice of parameters.

[6] We use data from the 1994 PSID data and wealth file. We included only households with strictly positive cash holdings in our sample. Money is defined as money in checking or savings accounts, money market funds, certificates of deposit, government savings bonds, and treasury bills.

[7] The number of productivity states $ne = 2$ is already found to generate sufficient heterogeneity in wealth and income.

Table 1
Parameterization of the OLG model

Preferences	β=0.99	σ=2	γ=0.981	η=7	η_0=0.26
Production	α=0.36	δ=0.019			
Market Structure	ϵ=6.0	ϕ=0.25			
Money Supply	π=1.013	ρ_θ=0.49	σ_θ=0.0089		
Government	ζ=0.3	a_0=0.258	a_1=0.768	a_2=0.031	

3.6 Computation

In order to compute business cycle dynamics of the model, we first need to compute the non-stochastic steady state of the model. Secondly, we log-linearize the model around the non-stochastic steady state.

The non-stochastic steady state is computed by solving the respective system of non-linear equations consisting of the first-order conditions of the generation born at time t, the government's budget constraint, and the aggregate consistency conditions. This is a system of several hundred variables (strictly speaking $(2(T + T^R - 1) + T)ne$ variables). We employ a non-linear equations solver that takes care of the admissible bounds within which the solution must lie. To obtain reasonable initial values, we started with a simplified version of our model, where it is easy to solve for the optimal time profile of the capital stock. We expanded this model in several steps to the model given above.

Thereafter, we log-linearize the model around the non-stochastic steady. This linear rational expectations model can be solved by, e.g., applying the method of Blanchard and Kahn (1980), (see King, Plosser, and Rebelo, 1988) or of King and Watson (2002).[8]

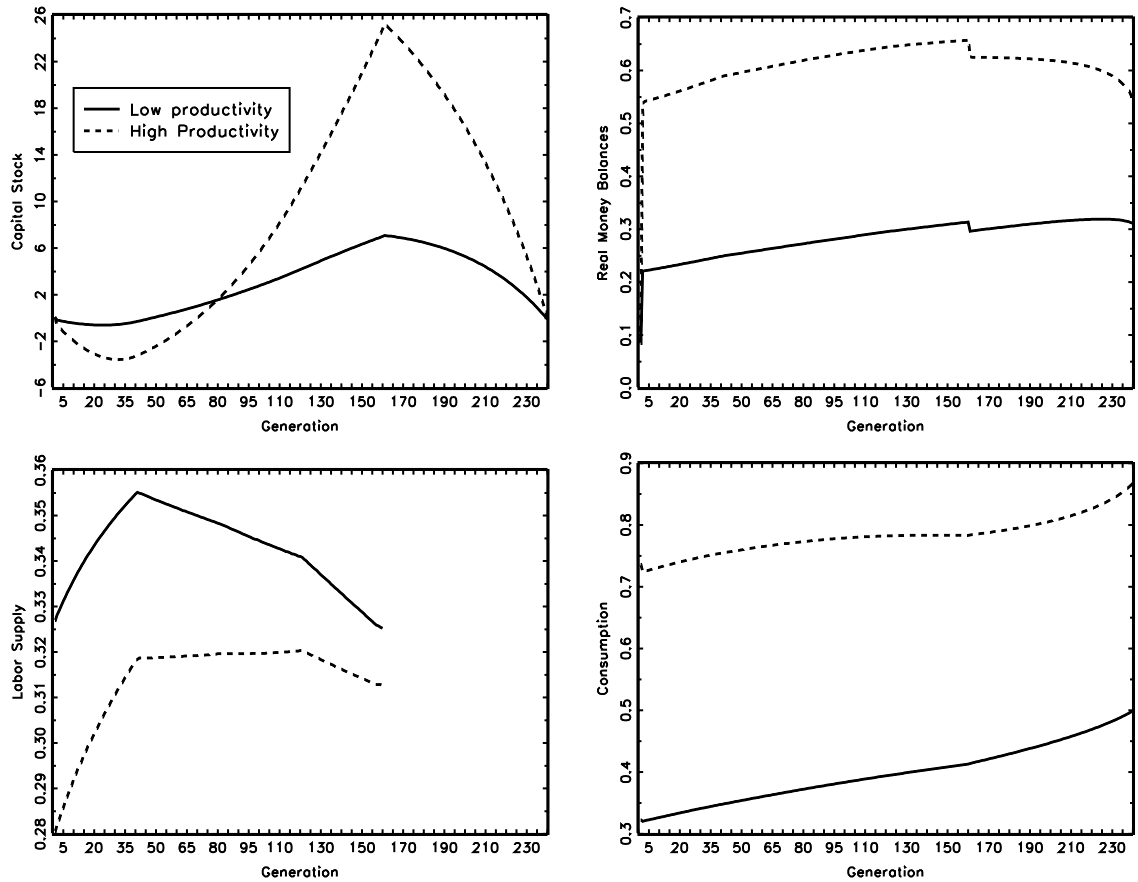
[8] The method that underlies our computation of the policy functions of the log-linearized model is explained in more detail in Chapter 2.3 in Heer and Maußner (2005) and applied to large scale dynamic systems of several hundred state variables and controls in Chapter 7.2.2. Both the Gauss code of the non-linear equations solver and the computation of the policy functions can be downloaded from the web side that accompanies Heer and Maußner (2005). The URL is `http://www.wiwi-uni.augsburg.de/vwl/maussner/dgebook/download.html`. The programs that solve the model are in `http://www.wiwi-uni.augsburg.de/vwl/maussner/englisch/chair/maussner/pap/demp.zip`.

4 Results

In this section, we present our findings on the redistributive effects of inflation. First, we present the behavior of the non-stochastic steady state. Second, we describe the dynamics of the distribution following a monetary shock.

Figure 1

Non-stochastic steady state



4.1 The non-stochastic steady state

In the non-stochastic steady state, inflation is constant. Figure 1 displays the consumption, savings and labor supply profiles for the 240 generations in our benchmark economy without government consumption. The wealth-age profile is hump-shaped as displayed in the upper left graph in Figure 1. Notice that due to the hump-shaped age-productivity profile (not displayed) households dissave during the first 61 quarters

(=15 years). Only at real lifetime age 35 do they start to build up positive savings. Agents with higher productivity attain higher levels of capital, money balances, and consumption. In addition, consumption as displayed in the lower right graph in Figure 1 is increasing over the life-time as the discount rate is smaller than the interest rate.[9]

Notice that the household behavior changes abruptly as they enter retirement. This kind of behavior is absent from most standard OLG models. Consumption growth increases at retirement, while there is a downward jump in the real money stock. The reason is the presence of progressive income taxation in our model. In the first period of retirement at age 60.25, taxable income falls and the tax rate on capital income is much smaller than during working life. For this reason, the after-tax rate of return on real capital income increases. As a consequence, consumption growth is higher, and the household readjusts its portfolio allocation. The premium on the return on capital relative to the one on money has increased, and the real money stock is reduced as can be seen from the upper right picture in Figure 1. For the same reason, money holdings are not proportional to consumption as usually observed in OLG models with money-in-the-utility and a CES utility function. As capital income and therefore the income tax rate falls during old age, retired workers decrease money holdings while consumption is even increasing. In our model, therefore, the correlation of consumption (and income) with money is lower than in standard DGE models with money-in-the-utility and in much better accordance with empirical observations.[10]

As can be seen from the lower left graph of Figure 1, labor supply does not attain a maximum prior to age 30 because the age-specific productivity is rather low at very young ages. Labor supply also attains its maximum prior to the maximum in the hourly wages because older agents have higher wealth and work fewer hours. In particular, high-productivity agents work less hours than agents with low productivity because their present discounted income exceeds the present discounted income of the poor workers.

In our economy, income and wealth are distributed unequally. The heterogeneity of

[9]In order to imply a more realistic consumption-age profile, we may have introduced stochastic survival probabilities; in this case, consumption declines at old age. However, our quantitative results are not sensitive to this modeling choice and, therefore, we kept the model as simple as possible.

[10]Heer, Maussner, and McNelis (2007) report a low correlation of income and real money for the US that is only averaging 0.22 in the years 1994, 1999 and 2001.

income is in good accordance with the one observed empirically. In particular, the Gini coefficient of total gross income amounts to 0.33 and the Gini coefficient of disposable income equals 0.26 in our model. For the US economy, Henle and Ryscavage (1980) estimate an average US earnings Gini coefficient for men of 0.42 in the period 1958-77, while Castañeda et al. (1998) report a Gini coefficient equal to 0.351. The distribution of wealth in our model is also close to the one observed empirically. In our model, the Gini coefficient of wealth amounts to 0.58 , whereas Greenwood (1983), Wolff (1987), Kessler and Wolff (1992), and Díaz-Giménez, Quadrini, and Ríos-Rull (1997) estimate Gini coefficients of the wealth distribution for the US economy in the range of 0.72 (single, without dependents, female household head) to 0.81 (nonworking household head).[11] Our model only fails to model the wealth concentration among the very rich agents. In order to replicate the wealth distribution of the top quintile, one had to introduce entrepreneurship as in Quadrini (2000).

4.2 Redistributive effects of inflation

An expansionary monetary shock increases demand. As prices are sticky and firms are monopolistic competitors in the intermediate goods sector, output and employment increases. The impulse response functions of aggregate variables to a monetary growth shock $\varepsilon_{\theta,2} = 1$ in period 2 (and zero thereafter) are presented in Figure 2 for the case $G_t \equiv 0$. In the first row, the percentage deviations of the variables money growth rate θ_t, output Y_t, consumption C_t, and investment I_t are graphed, in the second row, we illustrate the percentage deviations of effective labor input N_t and working hours (the dotted line), capital K_t, real money m_t, and the inflation factor π_t, while in the third row, you find the behavior of marginal costs g_t (the inverse of the mark-up), profits Ω_t, the real interest r_t, and the wage rate w_t.

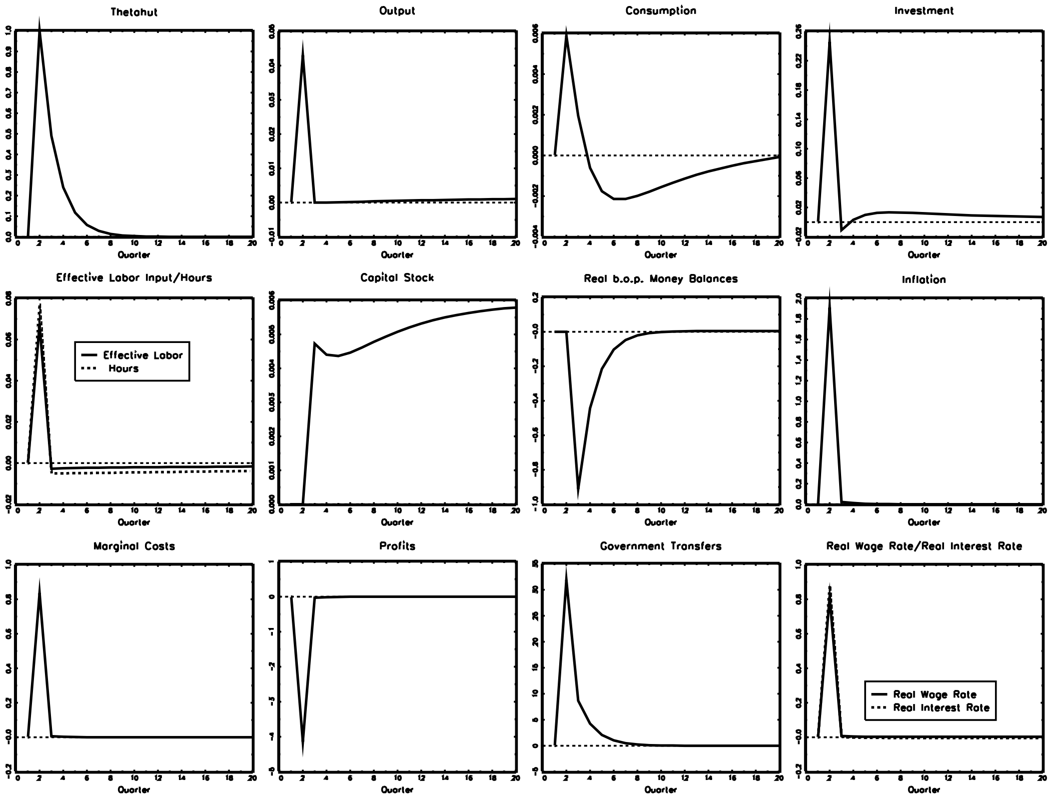
The percentage changes of output, hours and investment are small and only amount to 0.04%, 0.07% and 0.24%, respectively.[12] Inflation, the real interest rate, and wages all increase. Notice that, in this sticky price model, we are unable to model the liq-

[11] Huggett (1996) shows that we are able to replicate the empircally observable heterogeneity of wealth in a computable general equilibrium model if we introduce both life-cycle savings and individual earnings heterogeneity.

[12] The impulse response functions in the OLG model behave almost identical to those in the corresponding Ramsey model that we describe in the Appendix (see Figure 6). As one minor difference, we observe more consumption smoothing and a more persistent capital response in the OLG model.

Figure 2

Monetary shock in the OLG model



uidity effect that nominal interest rates decrease following an expansionary monetary policy. This is one of the major shortcomings of the sticky-price model that has been documented in the literature.[13] We, therefore, interpret our model carefully as a first step in order to get a full understanding of the redistributive effects of unanticipated inflation.[14]

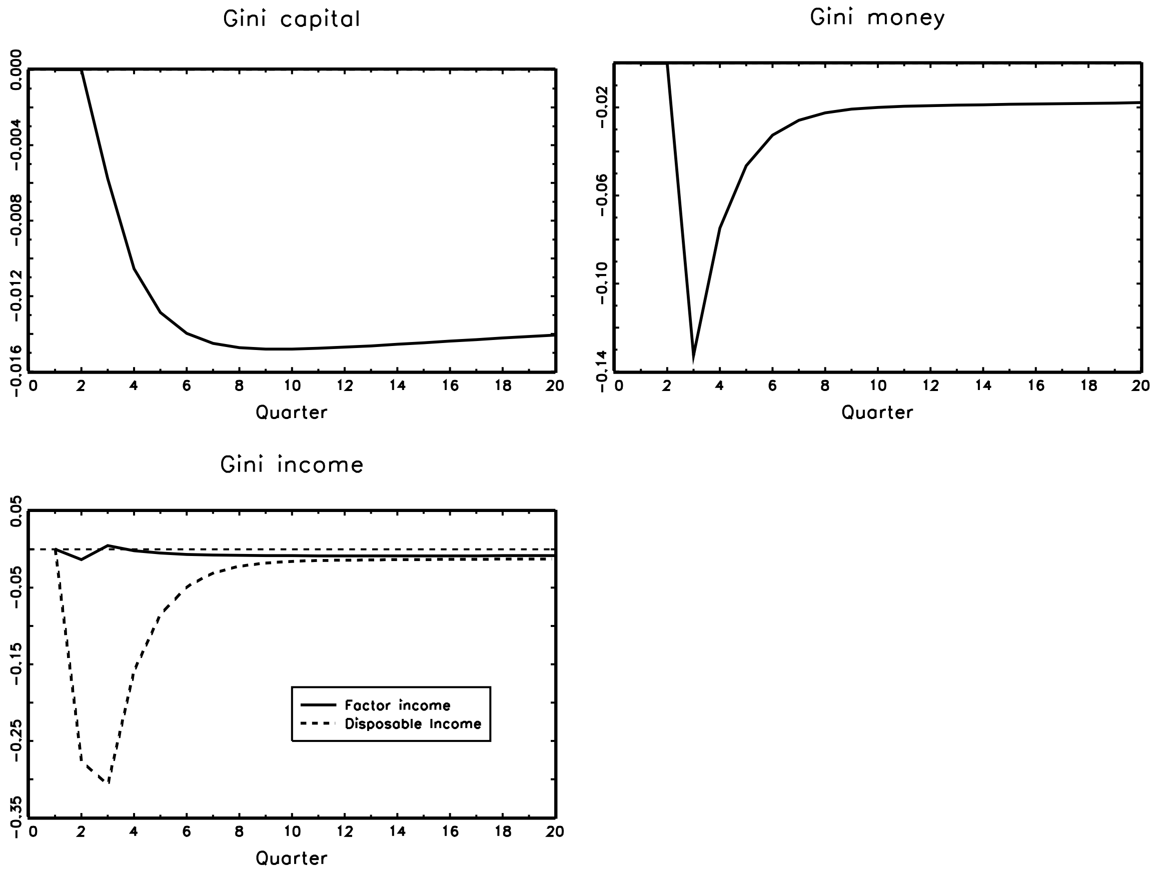
Following an unexpected increase of the money growth rate by 1% and a corresponding rise in inflation, the distribution of the factor income becomes a little less concentrated as illustrated in the lower right picture of Figure 3. The Gini coefficient of factor income (=pre-tax capital and labor income) declines, even though to a negligible extent of approximately 0.01%. Following a rise in real wages, the young and less productivite workers increase their labor supply a little more than the old and more productive

[13]See, among others, Christiano, Eichenbaum, and Evans (1997).

[14]In our model, we only consider the household's portfolio allocation on money and real-valued assets neglecting nominal fixed-income securities. Different from Doepke and Schneider (2006), we do not consider the redistributive effect of inflation from borrower to lender so that the missing liquidity effect is not a major impairment in our model.

Figure 3

Monetary shock in the OLG model and distribution



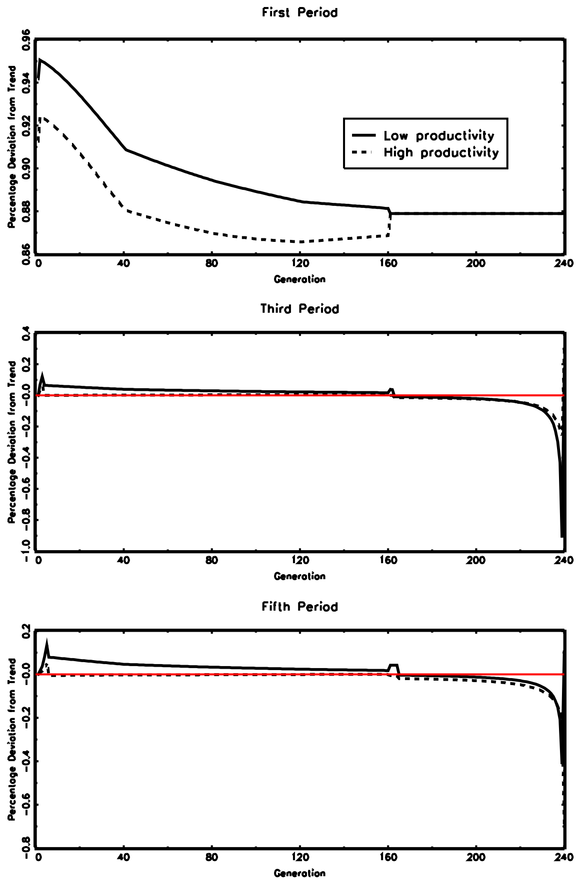
workers and wage income becomes less concentrated. Figure 4 displays the percentage change of the factor income of the 240 generations for the first 6 periods following the monetary shock. The young and low-productivity workers constitute the agents with low income and a corresponding low income tax rate. Therefore, their incentives to supply work increase by a larger extent than those of the income-rich agents. Disposable income that is defined as total income after taxes (including transfers, pensions and profit income) becomes even less concentrated than factor income. Even though pension income declines, this effect is compensated for by an increase of government transfers and higher marginal tax rates for the income-rich households. The Gini coefficient of disposable income declines by 0.3%.[15] We, therefore, do not observe that unanticipated

[15] If the government keeps transfers constant and rather increases government spending, disposable income becomes more concentrated. See Figure 7 in the Appendix.

inflation is hurting the poor due to its effects on factor prices and pensions as long as higher government revenues are distributed lump-sum.

Figure 4

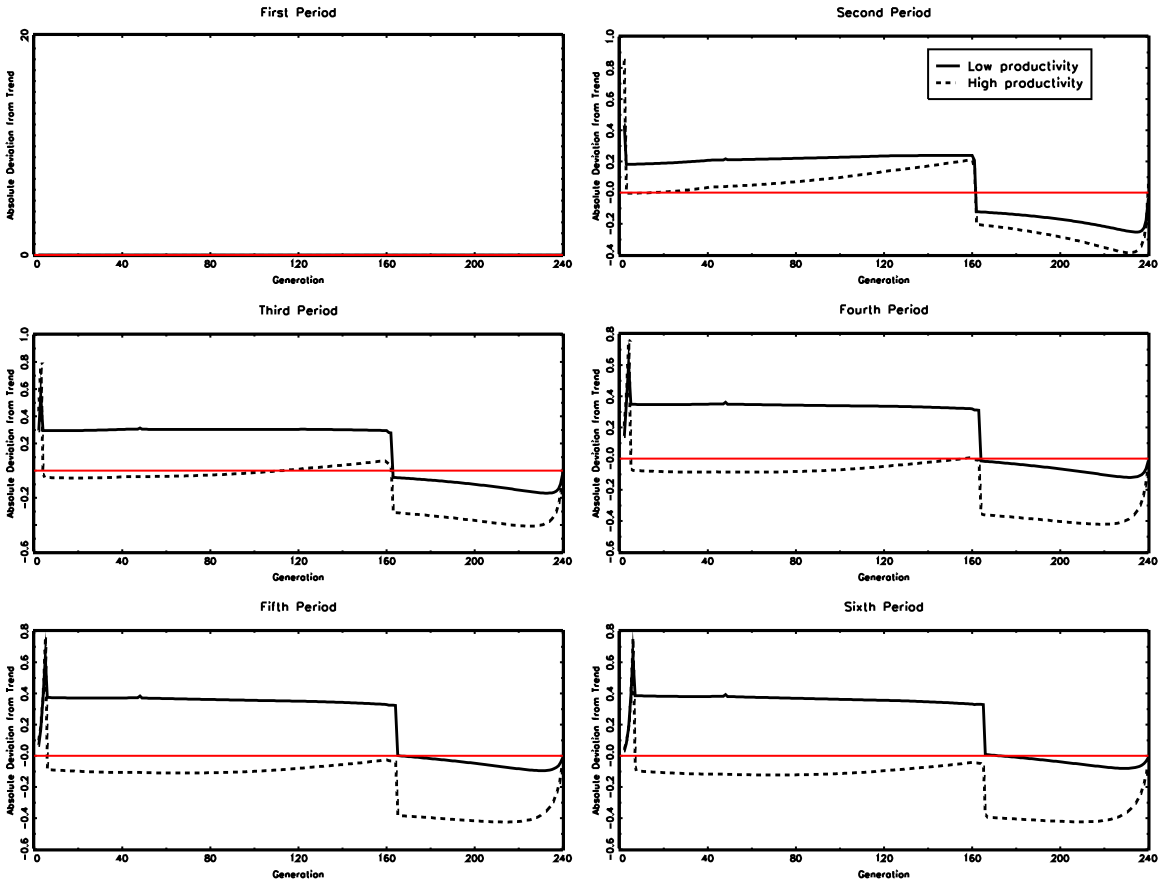
Monetary shock and factor income of the households



The changes of the disposable income is also reflected in a corresponding change of wealth inequality. As can be seen from inspection of Figure 3, the Gini coefficient of capital falls, even though the quantitative effect is negligible. The increase of the capital stock (in levels) for the individual generations and productivity types is graphed in Figure 5. All workers increase their savings in the period of the monetary expansion leading to an increase of the capital stock in the next period, while the retired agents reduce their savings due to lower pensions. In our model, the wealth-rich agents are clustered around the retirement age 60 corresponding to the model period 180. As young workers hold small or even negative wealth, the concentration of wealth declines subsequently, while the decrease in pensions of the relatively old retired workers has the opposite effect. The net effect is almost zero. If the government spends its additional tax revenues on consumption rather than on transfers, the concentration of wealth increases (not illustrated). Again, the quantitative effect is small as the Gini of capital increases by only 0.06%. We therefore conclude that inflation has a negligible effect on wealth inequality through its transmission channel of factor price changes and progressive taxation and mainly works through the redistribution from borrowers to lenders of fixed-income securities.

Figure 5

Monetary shock and changes in the individuals' capital stocks



5 Conclusion

Our study provides an initial step to the understanding of the distributional effects of monetary policy over the business cycle. So far, only the long-run distribution effects of monetary policy have been analyzed in computable general equilibrium models, as e.g. in Erosa and Ventura (2002), Cysne et al. (2005), or Heer and Süssmuth (2007). The effect of unexpected inflation on the distribution of income and wealth has not received attention in any dynamic general equilibrium model yet.

We present a model that replicates the following important channels of monetary policy on the distribution of income and wealth: 1) the response of prices, and hence the change in the mark-ups, interest rates, wages and, ultimately, the factor incomes of the individuals, 2) the 'bracket creep' effect, and 3) inflation-dependent pensions. In our model, an expansionary monetary shock is found to decrease the inequality of the distribution of factor income, even though only to a small extent. Depending on the tax and transfer system, the distribution of income and capital may even become more equal. For the transmission channels of inflation that we emphasize in our paper, we do not find support for the hypothesis that inflation is hurting the poor in the very short run. If the inflation tax is spent on government consumption only, however, inequality increases.

Our framework can only be regarded as a first step to a fully-fledged analysis of the short-run distribution effects of monetary policy. We emphasize the role of cash holdings and neglect the distribution of nominal asset price positions, in particular nominal fixed-income bonds. As described by Doepke and Schneider (2006), young middle-class households are the main beneficiaries of unanticipated inflation as they are net nominal borrowers due to their mortgage debt. Nevertheless, our analysis can serve as a benchmark case for future work that may consider a more sophisticated model with additional assets besides money and capital, namely housing and nominal bonds.

6 Appendix

6.1 Non-stochastic steady state of the OLG model

In the stationary state state of the OLG model (constant money growth $\bar{\theta} \equiv 1$), the following equilibrium conditions hold:

1. $\pi = \bar{\theta}$
2. $g = \frac{\epsilon - 1}{\epsilon}$.
3. $r = g\alpha K^{\alpha-1} N^{1-\alpha} - \delta$
4. $w = g(1-\alpha) K^{\alpha} N^{-\alpha}$.
5. $\Omega = (1-g) K^{\alpha} N^{1-\alpha}$.
6. $seign = \frac{Seign_t}{P_t} = (\bar{\theta} - 1) \sum_{j=1}^{ne} \sum_{s=2}^{T+T^R} \frac{\mu(j)}{T+T^R} \frac{M}{P}$.

6.2 The log-linear OLG model

In our model there are $ne[2(T+T^R-1)]$ variables with given initial conditions: the capital and cash holdings of generations $s = 2, 3, \ldots, T+T^R$.[16] We summarize these in the vectors $\mathbf{k}_t^j := [k_t^{2,j}, k_t^{3,j}, \ldots, k_t^{T+T^R,j}]'$, and $\mathbf{m}_t^j := [m_t^{2,j}, m_t^{3,j}, \ldots, m_t^{T+T^R,j}]'$, $m_t^{s,j} := M_t^{s,j}/P_{t-1}$, $j = 1, 2, \ldots, ne$. In addition, there are $ne(T+T^R-1)+2$ variables that are also predetermined at time t. These variables are the $ne(T+T^R-1)$ Lagrange multipliers $\boldsymbol{\lambda}_t^j := [\lambda_t^{1,j}, \lambda_t^{2,j}, \ldots, \lambda_t^{T+T^R-1,j}]'$, $j = 1, 2, \ldots, ne$, the inflation factor π_t and marginal costs g_t. The initial values of these variables must be chosen so that the transversality conditions hold. For given vectors $\mathbf{x}_t := [\mathbf{k}_t^2, \mathbf{m}_t^2, \mathbf{k}_t^3, \mathbf{m}_t^3, \ldots, \mathbf{k}_t^{ne}, \mathbf{m}_t^{ne}]'$ $\boldsymbol{\lambda}_t := [\boldsymbol{\lambda}_t^1, \boldsymbol{\lambda}_t^2, \ldots, \boldsymbol{\lambda}_t^{ne}, \pi_t, g_t]'$ the model's equations determine the vector $\mathbf{u}_t$. The elements of this vector are

- consumption $\mathbf{c}_t := [c_t^{1,1}, \ldots c_t^{T+T^R,1}, \ldots, c_t^{1,ne}, \ldots, c_t^{T+T^R,ne}]'$,
- working hours $\mathbf{n}_t := [n_t^{1,1}, \ldots n_t^{T,1}, \ldots, n_t^{1,ne}, \ldots, n_t^{T,ne}]'$,

[16] Since we assume that the cash transfer to the newborn, M_t^{1j}/P_{t-1} remain unchanged, we can ignore these additional ne state variables.

- factor income $\mathbf{y}_t := [y_t^{1,1}, \dots y_t^{T+T^R,1}, \dots, y_t^{1,ne}, \dots, y_t^{T+T^R,ne}]'$,
- the rental rate of capital r_t, the real wage w_t, the aggregate capital stock K_t, effective aggregate labor input N_t, the beginning-of-period stock of real money balances m_t, aggregate transfers Tr_t, and aggregate profits Ω_t. Thus, $\mathbf{u}_t$ is a vector of $ne[2(T+T^R)+T]+7$ elements.

We seek a representation of our model in the form

$$C_u \hat{\mathbf{u}}_t = C_{x\lambda} \begin{bmatrix} \hat{\mathbf{x}}_t \\ \hat{\boldsymbol{\lambda}}_t \end{bmatrix} + C_z \hat{\theta}_t, \tag{23a}$$

$$D_{x\lambda} E_t \begin{bmatrix} \hat{\mathbf{x}}_{t+1} \\ \hat{\boldsymbol{\lambda}}_{t+1} \end{bmatrix} + F_{x\lambda} \begin{bmatrix} \hat{\mathbf{x}}_t \\ \hat{\boldsymbol{\lambda}}_t \end{bmatrix} = D_u E_t \hat{\mathbf{u}}_{t+1} + F_u \hat{\mathbf{u}}_t + D_z E_t \hat{\theta}_{t+1} + F_z \hat{\theta}_t, \tag{23b}$$

where the hat denotes percentage deviations from the non-stochastic steady state value of a variable.

We first derive the set of equations (23a). The log-linearized Euler equations (6) are[17]

$$\begin{aligned} \hat{\lambda}_t^{s,j} &= (\gamma(1-\sigma)-1)\,\hat{c}_t^{s,j} + (1-\gamma)(1-\sigma)\left[\hat{m}_t^{s,j} - \hat{\pi}_t\right], \\ &\quad s = 1, \dots, T+T^R, \;\; j = 1, \dots, ne., \\ \hat{m}_t^{1,j} &= 0 \;\forall j = 1, \dots, ne. \end{aligned} \tag{24}$$

The log-linearized Euler equations (9) are:

$$\begin{aligned} \eta \frac{n^{sj}}{1-n^{s,j}} \hat{n}_t^{s,j} + \frac{\tau''}{1-\tau'} y^{s,j} \hat{y}_t^{s,j} - \hat{w}_t &= \hat{\lambda}_t^{s,j} - \frac{\tau' + \tau'' y^{s,j}}{1-\tau'} \hat{\pi}_t, \\ s = 1, 2, \dots, T, \; j &= 1, 2, \dots, ne, \end{aligned} \tag{25}$$

where τ' and τ'' denote the first and second derivative of the tax function evaluated at $y^{s,j}$, respectively. The $ne(T+T^R)$ definitions of factor income yield:

$$\begin{aligned} 0 &= y^{1,j} \hat{y}_t^{1,j} - we(1,j) n^{1,j} \hat{n}_t^{1,j} - we(1,j) n^{1,j} \hat{w}_t, \\ &\quad j = 1, \dots, ne, \\ rk^{s,j} \hat{k}_t^{s,j} &= y^{s,j} \hat{y}_t^{s,j} - we(s,j) n^{s,j} \hat{n}_t^{s,j} - we(s,j) n^{s,j} \hat{w}_t - rk^{s,j} \hat{r}_t, \\ &\quad s = 2, \dots, T, \; j = 1, \dots, ne, \\ rk^{s,j} \hat{k}_t^{s,j} &= y^{s,j} \hat{y}_t^{s,j} - rk^{s,j} \hat{r}_t, \\ &\quad s = T+1, \dots, T+T^R, \\ &\quad j = 1, \dots, ne. \end{aligned} \tag{26}$$

[17] We will use the ne equations for generation $T+T^R$ later to eliminate $\hat{\lambda}_t^{T+T^R,j}$, which is a control rather than a costate variable.

The log-linearized budget equations for generation $s = T + T^R$ are:

$$\begin{aligned}
&c^{T+T^R,j}\hat{c}_t^{T+T^R,j} - (1-\tau')y^{T+T^R,j}\hat{y}^{T+T^R,j} - tr\hat{tr}_t - \Omega\hat{\Omega}_t \\
&= (1-\delta)k^{T+T^R,j}\hat{k}_t^{T+T^R,j} + (m^{T+T^R,j}/\pi)\hat{m}^{T+T^R,j} \\
&\quad - (\tau' y^{T+T^R,j} + pens + (m^{T+T^R,j}/\pi))\hat{\pi}_t, \\
&j = 1, \ldots, ne.
\end{aligned} \tag{27}$$

From the factor market equilibrium conditions (14) and (15) we obtain:

$$\begin{aligned}
\hat{w}_t + \alpha\hat{N}_t &= \alpha\hat{K}_t + \hat{g}_t, \\
\hat{r}_t + (\alpha-1)\hat{N}_t &= (\alpha-1)\hat{K}_t + \hat{g}_t.
\end{aligned} \tag{28}$$

From aggregate profits $\Omega_t = (1-g_t)N_t^{1-\alpha}K^\alpha$, we derive

$$\hat{\Omega}_t - (1-\alpha)\hat{N}_t = \alpha\hat{K}_t + (1-\epsilon)\hat{g}_t. \tag{29}$$

The aggregate consistency conditions (22c), (22b), and (22d), imply

$$\begin{aligned}
K\hat{K} &= \sum_{j=1}^{ne}\sum_{s=2}^{T+T^R}\frac{\mu(j)}{T+T^R}k^{s,j}\hat{k}_t^{s,j}, \\
N\hat{N} &= \sum_{j=1}^{ne}\sum_{s=1}^{T}\frac{\mu(j)}{T+T^R}n^{s,j}\hat{n}_t^{s,j}, \\
m\hat{m}_t &= \sum_{j=1}^{ne}\sum_{s=2}^{T+T^R}\frac{\mu(j)}{T+T^R}m^{s,j}\hat{m}_t^{s,j}.
\end{aligned} \tag{30}$$

Finally, the log-linearized budget constraint of the government (20) is given by:

$$\begin{aligned}
&\sum_{j=1}^{ne}\sum_{s=1}^{T+T^R}\frac{\mu(j)}{T+T^R}\tau' y^{s,j}\hat{y}_t^{s,j} - tr\hat{tr}_t = (1-\theta)(m/\pi)\hat{m}_t \\
&+ \left(-\sum_{j=1}^{ne}\sum_{s=1}^{T+T^R}\frac{\mu(j)}{T+T^R}\tau' y^{s,j} + (\theta-1)(\tilde{m}/\pi) + ((m-\tilde{m})/pi) - \frac{T^R}{T+T^R}pens\right)\hat{\pi}_t - \tilde{m}\hat{\theta}_t,
\end{aligned} \tag{31}$$

where $\tilde{m} = m - \sum_{j=1}^{ne}(\mu(j)/(T+T^R))m^{1,j}$.

Next we derive the set of equations (23b). We begin with the log-linearized budget equations of generation $s = 1$. From (5) we derive:

$$\begin{aligned}
&k^{2,j}\hat{k}_{t+1}^{2,j} + m^{2,j}\hat{m}_{t+1}^{2,j} + (\tau' y^{1,j} + (m^{1,j}/\pi))\hat{\pi}_t = (1-\tau')y^{1,j}\hat{y}_t^{1,j} + tr\hat{tr}_t + \Omega\hat{\Omega}_t - c^{1,j}\hat{c}_t^{1,j}, \\
&j = 1, \ldots, ne.
\end{aligned}$$

$$(32)$$

For generations $s = 2, \ldots, T$ the log-linearized budget equations (5) are:

$$\begin{aligned}
&k^{s+1,j}\hat{k}_{t+1}^{s+1,j} + m^{s+1,j}\hat{m}_{t+1}^{s+1,j} - (1-\delta)k^{s,j}\hat{k}_t^{s,j} - (m^{s,j}/\pi)\hat{m}_t^{s,j} + (\tau' y^{s,j} + (m^{s,j}/\pi))\hat{\pi}_t \\
&= (1-\tau')y^{s,j}\hat{y}_t^{s,j} + tr\hat{tr}_t + \Omega\hat{\Omega}_t - c^{s,j}\hat{c}_t^{s,j}, \\
&j = 1, \ldots, ne, \; s = 2, \ldots, T.
\end{aligned} \tag{33}$$

For generations $s = T+1, \ldots, T+T^R-1$ they are:

$$\begin{aligned}
&k^{s+1,j}\hat{k}_{t+1}^{s+1,j} + m^{s+1,j}\hat{m}_{t+1}^{s+1,j} - (1-\delta)k^{s,j}\hat{k}_t^{s,j} - (m^{s,j}/\pi)\hat{m}_t^{s,j} + (\tau' y^{s,j} + (m^{s,j}/\pi) + pens)\hat{\pi}_t \\
&= (1-\tau')y^{s,j}\hat{y}_t^{s,j} + tr\hat{tr}_t + \Omega\hat{\Omega}_t - c^{s,j}\hat{c}_t^{s,j}, \\
&j = 1, \ldots, ne, \; s = 2, \ldots, T.
\end{aligned} \tag{34}$$

The Euler equations (7) and (8) yield:

$$\begin{aligned}
&\hat{\lambda}_{t+1}^{s+1,j} - \hat{\lambda}_t^{s,j} - \beta r \frac{\lambda^{s+1,j}}{\lambda^{s,j}}\left(\tau' + \tau'' y^{s+1,j}\right)\hat{\pi}_{t+1} \\
&= \beta r \frac{\lambda^{s+1,j}}{\lambda^{s,j}}\tau'' y^{s+1,j}\hat{y}_t^{s+1,j} - \beta r \frac{\lambda^{s+1,j}}{\lambda^{s,j}}(1-\tau')\hat{r}_{t+1},
\end{aligned} \tag{35}$$

$$\begin{aligned}
&\frac{\beta}{\pi}\frac{\lambda^{s+1,j}}{\lambda^{s,j}}\hat{\lambda}_{t+1}^{s+1,j} - \hat{\lambda}_t^{s,j} - \left[1 + \left(1 - \frac{\beta}{\pi}\frac{\lambda^{s+1,j}}{\lambda^{s,j}}\right)[(1-\gamma)(1-\sigma)-1]\right]\hat{\pi}_{t+1} \\
&+ [(1-\gamma)(1-\sigma)-1]\left(1 - \frac{\beta}{\pi}\frac{\lambda^{s+1,j}}{\lambda^{s,j}}\right)\hat{m}_{t+1}^{s+1,j} \\
&= -\gamma(1-\sigma)\left(1 - \frac{\beta}{\pi}\frac{\lambda^{s+1,j}}{\lambda^{s,j}}\right)\hat{c}_{t+1}^{s+1,j}, \\
&j = 1, \ldots, ne, \; s = 1, \ldots, T+T^R-1.
\end{aligned} \tag{36}$$

Note that in the equations for $s = T + T^R - 1$ we must replace $\hat{\lambda}_{t+1}^{T+T^R,j}$ by the right hand side of (24) for $t+1$ and $s = T + T^R$. The remaining two equations are given by the New Keynesian Phillips curve equation (16),

$$\beta\hat{\pi}_{t+1} - \hat{\pi}_t + \frac{(1-\phi)(1-\beta\phi)}{\phi}\hat{g}_t = 0, \tag{37}$$

and the log-linearized definition of the aggregate beginning-of-period real stock of money $m_t := M_t/P_{t-1}$. Together with equation (17) this definition implies:

$$\hat{m}_{t+1} - \hat{m}_t + \hat{\pi}_t = \hat{\theta}_t. \tag{38}$$

The $ne[2(T+T^R]+T]+7$ equations (24) through (31) define $\hat{\mathbf{u}}_t$ for given $\hat{\mathbf{x}}_t$ and $\hat{\boldsymbol{\lambda}}_t$. The dynamics of the system is then determined from the $ne[3(T+T^R-1)]+2$ equations (32) through (38).

The log-linear system (23) is determined if $ne[2(T+T^R-1)]$ of its Eigenvalues are within the unit circle and if $ne(T+T^R-1)+2$ Eigenvalues are outside the unit circle. This condition holds in our calibration.

6.3 The representative agent model

In the representative-agent Ramsey model we are, of course, unable to model pensions and to differentiate between working hours and effective labor input. Everything else is unchanged.

The representative household maximizes his infinite life-time utility

$$\sum \beta^t u(c_t, M_t/P_t, 1-n_t)$$

subject to

$$\begin{aligned} k_{t+1} + \frac{M_{t+1}}{P_t} &= (1-\delta + r_t(1-\tau[(\pi_t/\pi)(w_t n_t + r_t k_t)]))k_t \\ &\quad + \frac{M_t}{P_t} + (1-\tau[(\pi_t/\pi)(w_t n_t + r_t k_t)])w_t n_t + tr_t + \Omega_t - c_t. \end{aligned}$$

His decision variables in period $t=0$ are M_1, k_1, c_0, and n_0.

In this model, there are two predetermined state variables, the stock of capital k_t and beginning-of-period real money balances

$$m_t := M_t/P_{t-1} \Rightarrow \frac{M_t}{P_t} = \frac{m_t}{\pi_t}, \pi_t := \frac{P_t}{P_{t-1}}. \tag{39}$$

Using these definitions, we can write the first-order conditions as follows:

$$\lambda_t = u_c\left(c_t, \frac{M_t}{P_t}, 1-n_t\right) = \gamma\left(c_t\right)^{\gamma(1-\sigma)-1}\left(m_t/\pi_t\right)^{(1-\gamma)(1-\sigma)}, \tag{40a}$$

$$\lambda_t = \beta E_t \lambda_{t+1}\left(1-\delta + r_{t+1}(1-\tau'[(\pi_t/\pi)(w_t n_t + r_t k_t)](\pi_t/\pi))\right), \tag{40b}$$

$$\lambda_t = \beta E\left[\frac{\lambda_{t+1}}{\pi_{t+1}} + \frac{u_{M/P}\left(c_{t+1}, \frac{M_{t+1}}{P_{t+1}}, 1-n_{t+1}\right)}{\pi_{t+1}}\right] \tag{40c}$$

$$= \beta E\left[\frac{\lambda_{t+1}}{\pi_{t+1}} + \frac{(1-\gamma)\left(c_{t+1}\right)^{\gamma(1-\sigma)}\left(m_{t+1}/\pi_{t+1}\right)^{(1-\gamma)(1-\sigma)-1}}{\pi_{t+1}}\right], \tag{40d}$$

$$u_n\left(c_t, \frac{M_t}{P_t}, 1-n_t\right) = \eta_0(1-n_t)^{-\eta} = \lambda_t w_t\left(1-\tau'[(\pi_t/\pi)(w_t n_t + r_t k_t)](\pi_t/\pi)\right). \tag{40e}$$

Non-stochastic steady state of the Ramsey model The stationary solution of the representative agent model is characterized by the following set of equations. Since real money balances are constant, the inflation factor π equals the money growth factor θ:

$$\pi = \theta. \tag{41a}$$

Calvo price staggering implies

$$g = \frac{\epsilon - 1}{\epsilon}. \tag{41b}$$

The stationary version of the Euler equation for capital,

$$\frac{1-\beta(1-\delta)}{\alpha\beta g} = n^{1-\alpha}k^{\alpha-1}\tau'[gn^{1-\alpha}k^{\alpha}]$$

can be solved for k given our predetermined value of $n = 0.33$. Given the solution for k we can determine y. The stationary version of the economy's resource constraint,

$$y = c + \delta k \tag{41c}$$

allows us, then, to compute c. Finally, the Euler equation (40c) implies the stationary solution for the ratio between consumption and real money balances:

$$\frac{C}{M/P} = \frac{\gamma}{1-\gamma}\left[\frac{\mu}{\beta} - 1\right]. \tag{41d}$$

We use this equation and (41c) to determine the value of γ. This is all we need to compute the policy function of the log-linearized model.

Calibration of the Ramsey model In this model we calibrate the tax function so that the marginal tax rate paid by the representative household in the non-stochastic steady state equals the marginal tax rate on the average US-income. The government's tax revenues are transferred lump-sum to the representative agent. Capital's share is $\alpha = 0.36$ and δ equals 0.019, as is the case in the OLG model. The parameters that determine the properties of the productivity shock and the money supply shock are the same as those used in the simulations of the OLG model. The remaining parameters are set as follows: β, γ and η_0 are chosen so that

- the annualized capital-output ratio is the same in both models (i.e., $K/Y = 2.1$)
- the representative agent works $n = 1/3$ hours,
- the velocity of M1 is the same in both models (i.e., $Y/(M/P) = 1.5$)

The log-linear Ramsey model The log-linear version of (40a) is given by

$$[\gamma(1-\sigma)-1]\hat{c}_t = -(1-\gamma)(1-\sigma)\hat{m}_t + \hat{\lambda}_t + (1-\gamma)(1-\sigma)\hat{\pi}_t. \tag{42a}$$

Log-linearizing (40e) delivers:

$$\eta\frac{n}{1-n}\hat{n}_t - \hat{w}_t + \frac{\tau'' gy}{1-\tau'}\hat{y}_t = \hat{\lambda}_t - \frac{\tau' + \tau'' gy}{1-\tau'}\hat{\pi}_t - \frac{\tau'' gy}{1-\tau'}\hat{g}_t, \tag{42b}$$

where τ' (τ'') is the marginal tax rate (the second derivative of the tax function) computed at the steady state solution of $gy = wn + rk$. The cost-minimizing conditions (15) and (14) provide two additional equations:

$$\alpha\hat{n}_t + \hat{w}_t = \alpha\hat{k}_t - \hat{x}_t, \tag{42c}$$

$$(\alpha-1)\hat{n}_t + \hat{r}_t = (\alpha-1)\hat{k}_t - \hat{x}_t. \tag{42d}$$

The log-linear version of the aggregate production function is given by:

$$(\alpha-1)\hat{n}_t + \hat{y}_t = \alpha\hat{k}_t. \tag{42e}$$

The definition of gross investment $i_t = y_t - c_t$ implies

$$[(y/i)-1]\hat{c}_t - (y/i)\hat{y}_t + \hat{i}_t = 0. \tag{42f}$$

Finally, the profit equation $\Omega_t = y_t(1-g_t))$ provides the following log-linear equation:

$$-\hat{y}_t + \hat{\Omega}_t = (1-\epsilon)\hat{g}_t. \tag{42g}$$

The five equations that determine the dynamics of the log-linear model are derived from the economy's resource constraint $k_{t+1} = (1-\delta)k_t + y_t - c_t$, the Euler equations for capital and money balances, (40b) and (40c), the definition of beginning-of-period money balances (39), and from the Calvo price staggering model:

$$\hat{k}_{t+1} + (\delta - 1)\hat{k}_t = (y/k)\hat{y}_t - (c/k)\hat{c}_t, \tag{43a}$$

$$\begin{aligned} E_t\hat{\lambda}_{t+1} - \hat{\lambda}_t - \beta r\frac{\tau' + \tau'' gy}{1-\tau'}E_t\hat{\pi}_{t+1} & \\ -\beta r\tau'' gyE_t\hat{g}_{t+1} & \\ &= \beta r\tau'' gyE_t\hat{y}_{t+1} - \beta r(1-\tau')E_t\hat{r}_{t+1}, \end{aligned} \tag{43b}$$

$$(\beta/\pi)E_t\hat{\lambda}_{t+1} - \hat{\lambda}_t - \Delta_1 E_t\hat{\pi}_{t+1} + \Delta_2 E_t\hat{m}_{t+1} = -\Delta_3 E_t\hat{c}_{t+1}, \tag{43c}$$

$$E_t\hat{m}_{t+1} - \hat{m}_t + \hat{\pi}_t = \hat{\theta}_t, \tag{43d}$$

$$\beta E_t\hat{\pi}_{t+1} - \hat{\pi}_t + \frac{(1-\phi)(1-\beta\phi)}{\phi}\hat{g}_t = 0, \tag{43e}$$

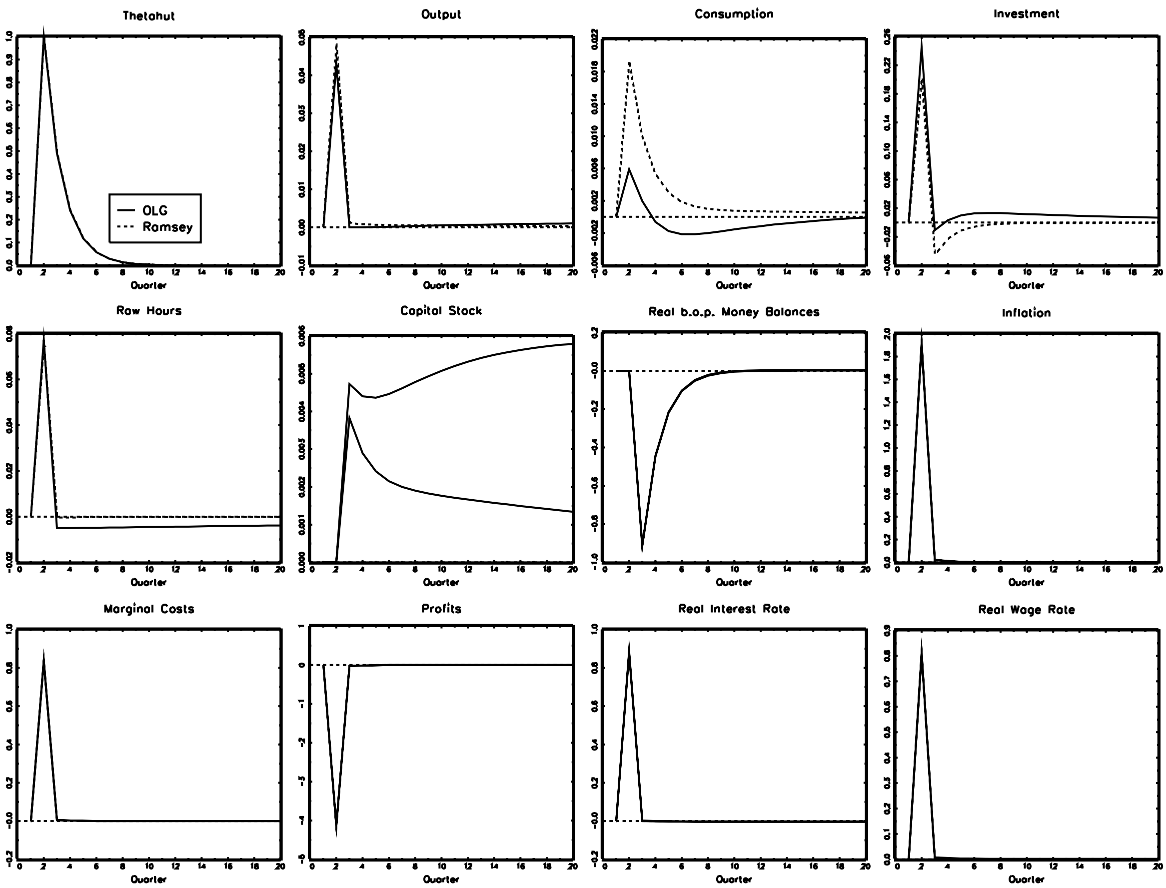
$$\begin{aligned} 1 + (1-(\beta/\pi))[(1-\gamma)(1-\sigma) - 1] &=: \Delta_1, \\ \Delta_1 - 1 &=: \Delta_2, \\ (1-(\beta/\pi))\gamma(1-\sigma) &=: \Delta_3. \end{aligned}$$

6.4 Dynamics in the corresponding representative agent model

The impulse response functions of the aggregate variables in the Ramsey model with Calvo price staggering are graphed in Figure 6 (the dotted lines). Again the ordering of the variables is identical to the one in Figure 2. Notice that the qualitative behavior of the variables in response to a monetary expansion is the same in the two economies for all variables with a minor exception. In the OLG model, there is a little more consumption smoothing than in the representative-agent economy so that the capital stock remains above its non-stochastic level for many quarters.

Figure 6

Monetary shock in the representative-agent economy

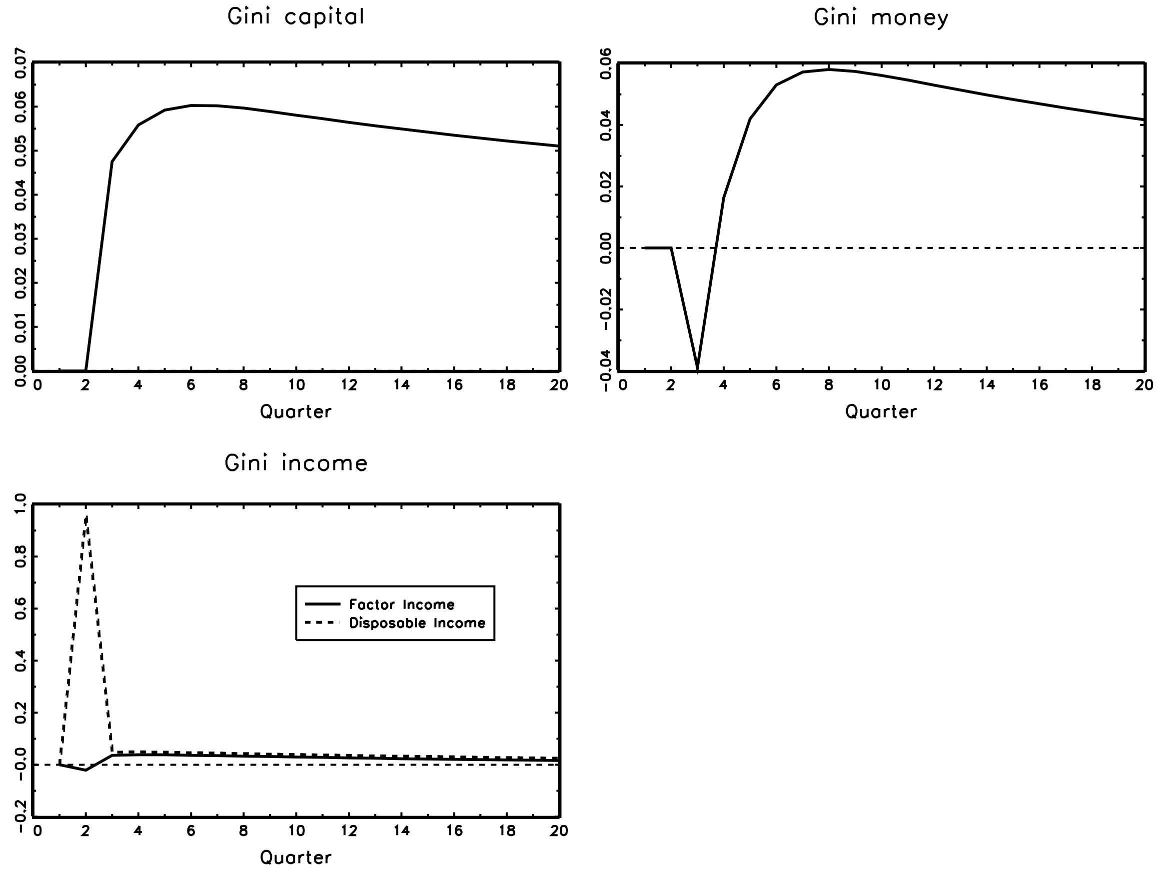


6.5 OLG model with government consumption

In Figure 7, the percentage responses of the Gini coefficients of capital, money, and income are graphed for the model with government consumption. In this model we increased the discount factor from $\beta = 0.99$ to $\beta = 0.9909$ and decreased γ from 0.981 to 0.98 so that both the after tax real rate of return and the velocity of money are the same as in our benchmark solution. As government transfers do not change (and remain equal to a small constant), disposable income increases by a smaller extent for the young and low-productivity workers than in the benchmark case.

Figure 7

Monetary shock in the OLG model with government consumption and distribution



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