

RC Building Modelling for Control Purposes: A Case Study

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Abstract

When dealing with models, a key factor to consider when selecting their features is the context in which the models will be used: for example, they could be used for design or for control purposes. If we focus on the second case, the model should be accurate enough to capture the principal dynamics of interest and simple enough to minimize the computational effort. In building modelling for control, a promising paradigm seems to be the use of simplified grey-box models. This paper presents a case study in which the existing temperature control strategy can be improved with the resulting possibility of considerable energy saving. More in detail, we introduce here the first step of the entire process: the choice of the model of the system. We decided to investigate the use of a grey-box model, the parameters of which were estimated using a parametric identification process. Thanks to this approach, full knowledge of the system is not required but this lack of information needs to be balanced with the use of measured data. We decided to use only measured data during the standard operation mode of the system for the parameter identification process. Thus we did not perform targeted experiments on the real system, because of all the restrictions in the specific context. Using this approach, it was still possible to achieve good results in terms of deviation between model simulation and data (indoor air: RMSE = 0.31 and $R^2 = 0.92$).

1. Introduction

Buildings account for 20–40 % of the total energy consumptions (Parry et al., 2007) and, as a result, in the past decades, great efforts have been made in trying to reduce this data.

When dealing with energy efficiency in buildings, the main research areas focus on retrofitting and modernization. On top of these, a promising

approach is the application of advanced control strategies in the building automation systems (BAS).

The potentiality of this approach is shown, for example in (Liao and Dexter, 2004): the only improvement of a boiler control can lead to an energy saving up to 20 %.

Despite the research efforts in improving advanced control techniques, the most widely used approach for temperature control in buildings, is the use of a single heating-curve, tuned according to the climatic zone and few building characteristics, and a feedback temperature control. In some cases, in addition to the heating-curve, the heating, ventilation and air conditioning system (HVAC) is locally controlled with rule-based controllers (RBC) that use an “if-then-else” strategy to maintain the desired ambient condition in each room. With this control configuration, what is immediately evident is the lack of an optimized strategy for the entire building control. Advanced control techniques can overcome this problem.

The main research directions in the above-mentioned field are: 1) Learning based approaches like neural networks, genetic algorithms, fuzzy techniques, etc.; 2) Model predictive control (MPC).

With reference to the first research field, an interesting review about neural network in building can be found in (Kumara et al., 2013), whereas for MPC in buildings, a review can be found in (Prívvara et al., 2012).

The first class of methods needs a lot of data from the real system, and on-going learning is required to face with changes of different nature, like changes in the physics of the buildings but also in the occupancy behavior.

On the contrary, building models for MPC are generally more linked to the physics of the buildings, e.g. (Bacher and Madsen, 2011), thus it is easier to deal with changes in the building structure or with the inputs. In addition, with this approach, it is easy to create scalable models for the optimization process and deal with constrained problems. As will be discussed in the next sections of this work, the second class of methods is chosen because of the purpose of the work.

The present work is, in fact, part of a larger study devoted to energy saving in public buildings. More in detail, the final aim of the work is to achieve energy saving using different control strategies.

Reading the previous text, the importance of modelling as a crucial part for advanced control strategies, can be inferred. In this paper, the chosen modelling approach will be discussed, and the results of the model tuning process will be presented and treated. It is worth remembering that the goal of the modelling step is to create a simple, but accurate enough model that can be used for control purposes.

2. Mathematical Approach

When dealing with mathematical models, two general classes can be detected: *forward* and *inverse* models (ASHRAE, 2001). Models belonging to the first class are based only on the physical knowledge of the system. This kind of approach is mainly used for design because one does not need to observe an existing system to create a model. For example, in building modelling, the design of a HVAC system can be performed by taking into account the desired behavior of the system and the physical characteristics of the involved elements.

Conversely, the inverse modelling approach is primarily used for performance monitoring, control system design, and application of on-line control strategies. This because an inverse model needs data to be tuned and it thus requires a real system.

It is worth noticing that both forward and inverse modes result in a set of equations with parameters, but the main difference between the two approaches is in how to define the values of said parameters.

For example, suppose that we need to set the value of the specific heat of a wall. In forward modelling

framework, this value is derived directly by the wall material knowledge. Conversely, in inverse modelling, the same parameter is set to its values using data collected during an appropriate experiment on the wall thermal properties, without any wall material information.

Therefore, it is clear that the second approach is particularly attractive if one has limited knowledge of the system and a big amount of data.

As said before, we decided to investigate the second approach, so the information contained in the data is used to define the value of the model parameters.

2.1 Grey-Box Model

The model chosen here to describe the building structure belongs to the inverse class just presented but more in detail, to the grey-box models class.

A grey-box model combines partial a-priori knowledge of the system with empirical knowledge obtained by data. Particularly, the first type of knowledge is used to define the structure of the building model and the second to tune parameters. This kind of tuning via data usage is called *parameter identification* and it will be discussed more in detail in the next session.

We decided to use grey-box modelling because we know the building in terms of the main characteristics but we do not know in detail all the physical elements. Through the correct use of data, we can overcome this lack of knowledge.

The building model structure results in a set of differential equations with some parameters and we decide to represent them with a simple electric equivalent, through an R-C network representation. This kind of approach is well explained for example in (Parnis and Sproul, 2010) and it is based on the use of electrical components to represent thermal quantities.

2.2 Parameter Identification: Methodology and Specific Issues

As said before, parameter identification is used to define the values of the model parameters.

This method is based on the use of a set of relevant data from a real system. This set must be divided into training and testing data. The first set is then used to perform parameter identification, while the

second is used to verify the validity of the tuned model. One of the key factors of this approach is the use of appropriate data.

In particular, during the data collection, the input to the real system must be *persistently exciting* (Bittanti, 2005). This results in the fact that all the dynamics of the system are excited by the input signal, namely in the trend of the output variable there is all the information to obtain the value of the system parameters.

Often, an experiment performed on the system is required to respect this fundamental rule of parameter identification, but, in some cases, it is not trivial to create an input to the system with the desired characteristics. In the building framework, for example, there are a lot of restrictions.

In particular, in our case study, the first restriction is caused by the configuration of the plant. What we want to handle is the power input from the heating system, but on the real plant this variable cannot be directly changed. Therefore, we need to act on other variables (e.g. valve openings) through the control unit, thus we have to face all the restrictions due to the control unit configuration and operation mode. The second important issue is linked to the inner conditions of the building. If the thermal power entering the building has the needed characteristics, the indoor air temperature fluctuations would be too large to maintain the indoor ambient comfort conditions.

These problems can be partially overcome with the choice of an appropriate period during the year in which the principal dynamics of the building are excited enough with the normal behaviour of the plant and with the other external inputs.

As said before, the parameter identification process is used to define parameter values through data usage. The mathematical problem associated to this idea is an optimization problem, the goal of which is to find the minimum of an appropriate objective function (Nocedal and Wright, 1999).

In parameter estimation, dealing with deterministic models, the most commonly used approach is to set a least-squares problem. This means that the objective function is in the form:

$$f(x) = \sum_{i=1}^n (ym_i - y_i)^2 \quad (1)$$

where ym_i is the measure of the output and y_i is the output of the model. This means that the distance between the measure and the output of the model has to be as small as possible compatibly with the model structure.

The only way to minimize the objective function is to change the model parameters in the proper way so as to change the output variable y_i and thus the objective function.

3. Case Study

We prove the validity of our approach on the basis of a real building located in Lombardy (Northern Italy), a public structure used as a primary school. The building hosts four classrooms and a canteen with a big hall in the middle. There is an underfloor heating system and no feedback control. The regulation of the heating system is performed with the use of a heating-curve, thus using the external temperature, and on the basis of time scheduled operating modes.

We decided to collect the needed data through a small set of non-invasive sensors. We used indoor temperature data logger to measure the indoor temperatures and decided to estimate the underfloor heating water temperature using PT1000 sensors, connected to embedded data collectors, on pipe surfaces.

To measure the solar radiation, a small climate station with a pyranometer was also placed near the building. The external temperature data are collected using the sensors already existing on site and used for the plant regulation. We also used PT1000 sensors to measure the flow and return water temperature of the secondary circuit.

Because of both the distribution system configuration and the control strategy, we could avoid the use of a permanent flowmeter on the secondary circuit. Namely, the heating water is always flowing in the secondary circuit and the circuit does not change its configuration.

The mass flow was thus evaluated using a portable ultrasonic flow measuring system installed for a relatively short period.

In Table 1, the accuracy of the instruments, according to the datasheets, are given.

Table 1 – Measurement equipment accuracy

Instrument	Accuracy
Indoor temperature data logger	$\pm 0.35 \text{ }^\circ\text{C}$
Outside temperature sensor	$\pm 0.2 \text{ }^\circ\text{C}$ influence of temperature <10 $^\circ\text{C}$, >40 $^\circ\text{C}$ --> $\pm 0.007 \text{ }^\circ\text{C}/^\circ\text{C}$
Pt1000	$\pm 0.15 \pm 0.002 T \text{ }^\circ\text{C}$
Pyranometer	Second class (ISO, 1999)
Flowmeter	$\pm 2 \%$ o.r. $\pm 7.5 \text{ mm/s}$

The present study is based on data collected during the winter 2014–2015.

The following time series were collected with the measurement equipment: T_{in} , T_{out} , T_{ext} , Φ_r .

In addition to the above time series, T_a , namely the mean value (weighing on volumes) of the measured temperatures inside the building, is computed and used for the parameter identification process.

On the basis of the available data and on the structure of the building, we selected a set of possible model structures.

In the present work, we only show the most appropriate one on the basis of the result analysis.

As said before, the chosen model can be represented with an electric equivalent: in Fig. 1 the RC-network of the model is shown.

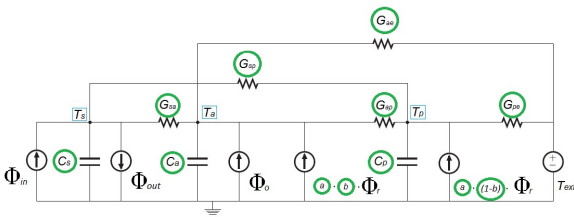


Fig. 1 – RC-network of the building model

The model has one manipulable input variable, i.e. the thermal power entering the building through the heating system, Φ_{in} , and three non-manipulable inputs: the solar global irradiance, Φ_r , the internal gain provided by the occupants and the internal supplies, Φ_o , and the external temperature, T_{ext} .

It is important to notice that the thermal input from

the plant is divided into power entering the building, Φ_{in} , and power exiting the building, Φ_{out} , thus following the water flows.

There are three state variables, named T_a , T_s and T_p , which represent respectively the indoor air temperature, the floor temperature and the external walls temperature (included the roof). Each state variable is associated with the corresponding heat capacity, namely C_a , C_s and C_p .

Each G_{xy} term represents the thermal conductance between generic temperatures T_x and T_y .

Φ_o input is computed only according to the occupation schedule and with a fixed thermal coefficient per person.

The parameter a is used to scale the solar input, which is already weighted on the basis of the external area of the building, while b , is used to share the solar input into two terms: one affecting on the walls and the other directly on the indoor air temperature. This second term ideally represents the portion of the solar input entering the building through the windows. The main assumption, which supports the last statement, is that the thermal input entering the windows, and thus affecting the floor, is totally transferred to the air node because of the insulation of the floor.

In Fig. 1, the powers entering and exiting the building (Φ_{in} and Φ_{out}), are used, but, given that

$$\Phi_{in} = \dot{m} \cdot c \cdot T_{in} \tag{2}$$

$$\Phi_{out} = \dot{m} \cdot c \cdot T_{out} \tag{3}$$

and that the term $\dot{m} \cdot c$ can be considered as a constant, it can be convenient to use T_{in} as an input variable and T_{out} as output.

Another important assumption, is the chosen relation between T_{out} and the other variables.

So the following equation was derived from the stationary model of the heat exchange along a pipe:

$$T_{out} = \alpha \cdot T_{in} + (1 - \alpha) \cdot T_s \tag{4}$$

with $0 < \alpha < 1$.

In Eq. (5) the state-space representation of the system is given.

$$\begin{cases} \frac{dx}{dt} = Ax + Bu \\ y = Cx + Du \end{cases} \tag{5}$$

where:

$$x = \begin{pmatrix} T_s \\ T_a \\ T_p \end{pmatrix}, \quad y = \begin{pmatrix} T_a \\ T_{out} \end{pmatrix}, \quad u = \begin{pmatrix} T_{in} \\ \Phi_o \\ \Phi_r \\ T_{ext} \end{pmatrix}$$

$$A = \begin{bmatrix} a_1 & G_{sa}/C_s & G_{sp}/C_s \\ G_{sa}/C_a & a_2 & G_{ap}/C_a \\ G_{sp}/C_p & G_{ap}/C_p & a_3 \end{bmatrix}$$

$$a_1 = -(G_{sa} + G_{sp} + \dot{m} \cdot c \cdot (1 - \alpha))/C_s$$

$$a_2 = -(G_{sa} + G_{ap} + G_{ae})/C_a$$

$$a_3 = -(G_{ap} + G_{sp} + G_{pe})/C_p$$

$$B = \begin{bmatrix} b_1 & 0 & 0 & 0 \\ 0 & 1/C_a & a \cdot b/C_a & G_{ae}/C_a \\ 0 & 0 & a \cdot (1 - b)/C_p & G_{pe}/C_p \end{bmatrix}$$

$$b_1 = \dot{m} \cdot c \cdot (1 - \alpha)/C_s$$

$$C = \begin{bmatrix} 0 & 1 & 0 \\ 1 - \alpha & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \alpha & 0 & 0 & 0 \end{bmatrix}$$

Here, the output variables are T_a and T_{out} .

The use of T_a as output variable is a common practice because the focus is on the indoor condition of the building. The use of T_{out} as output is closely associated to the plant configuration and the future use of the model. This variable is indeed the link between the plant and the building models. Thus, it is very important that the trend of this variable is well represented in the model simulation.

In addition to the choice of the model outputs, it is also crucial to underline some aspects about model parameters.

In Fig. 1, the parameters involved in the identification process are circled in green. As said before, all G_{xy} and C_x terms have a physical meaning so it is worth noticing that the initialization values for the identification process can be chosen taking into account the physical knowledge of the system.

Because of the stated aim to simplify as much as possible the model structure and the consequent use of a lumped approach, these parameters included a lot of different physical elements. Therefore, the guess values for the identification process were chosen based on a generic knowledge of the system. For example, we used a single element to define a generic external wall without difference between ceiling, floor and exterior walls. This simplification led us not to consider each single layer of the wall element with the related features (thickness, specific

heat, density, etc.) but only average characteristics of the generic wall.

4. Results and Discussions

In this section, the main results are presented.

First, the results of the parameter identification process are shown and then the model validation is presented and discussed.

4.1 Identification Results

In the identification process, the focus is on the correct representation of the output variables, those involved in the minimization process.

Therefore, the first check on the results has to be performed considering the difference between data and model outputs.

In Fig. 2 and Fig. 3, the difference between simulation results (red lines) and data (blue lines) are shown for the internal ambient temperature and the return water temperature respectively.

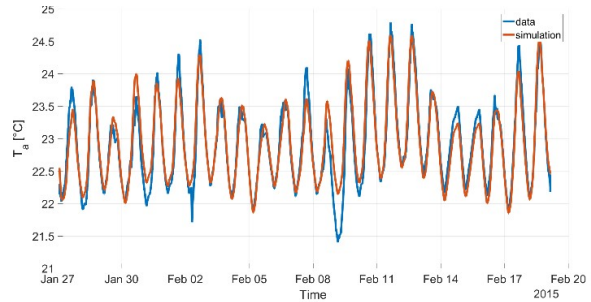


Fig. 2 – Simulation VS tuning data: indoor air temperature

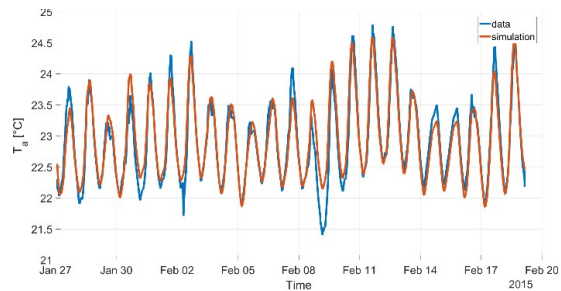


Fig. 3 – Simulation VS tuning data: return water temperature

First, it is important to analyse the simulation result dealing with the ambient temperature because the model is primarily devoted to the evaluation of the inner conditions. If we analyse Fig. 2, we can see that the maximum deviation between measurement and

simulation (0.78 °C) occurs during 09/02. Despite the fact that such a deviation could be too big for our purposes, it is necessary to understand what could be the cause of such a big difference between the data and the model simulation results.

With a good probability, this discrepancy could be due to an unpredictable disturbance on the ambient temperature. The opening of a window represents an example of this kind of disturbances. It happens in an arbitrary manner depending on the needs of the building occupants, we do not have sensors to measure this event, and it causes an abrupt decrease in temperature. If the opening continues for a considerable time, air temperatures can deviate very significantly from the simulated temperature as seems to occur on 09/02. To confirm this hypothesis, we can analyse the return water temperature, Fig. 3, in the same period. What we can immediately notice is that the simulation does not differ clearly from the measured data. This is because the variation of air temperature in the data is not due to the effective cooling of the building, but to a phenomenon that acts directly on the internal temperature.

We decided not to model this phenomenon because it is not very frequent and requires some sensors to be detected; thus, it is obvious that the model differs from the data when it occurs.

In Fig. 3, the second output variable, the return temperature, is shown.

The simulation result is slightly better than the previous one: the simulation deviates from the given measured by less than 0.5 °C and the dynamic appears to be well represented with the identified model.

Once verified the good agreement of the time series, it is also important to quantify the results using some standard performance indexes.

Therefore, in Table 2, the RMSE and R^2 values are listed in order to quantify T_a and T_{out} deviation from the data.

Table 2 – Model deviation from identification data

Index	T_a	T_{out}
RMSE	0.21	0.16
R^2	0.89	0.98

Considering the results shown in Table 2, we can confirm that the model accuracy in reproducing the return water temperature is higher if compared to the indoor air representation.

This result is not surprising if we remember that the identification process is performed using an average room temperature.

When dealing with grey-box models, as in this case, it is also possible to analyse the identification results considering the meaning of the parameters.

In Table 3, the identification results, in terms of parameter values, are listed.

For a better understanding of the numerical results, the thermal conductance and capacities are reported as a ratio of the total air volume.

Table 3 – Parameter values

Parameter	Value	Unit
Gsa	0.924	W/(K · m ³)
Gsp	0.596	W/(K · m ³)
Gap	1.481	W/(K · m ³)
Gpe	0.444	W/(K · m ³)
Gae	0.219	W/(K · m ³)
Cs	26.280	kJ/(K · m ³)
Ca	43.283	kJ/(K · m ³)
Cp	116.108	kJ/(K · m ³)
a	0.727	-
b	0.618	-
α	0.176	-

To evaluate this result, it is important to remember that we use a lamped model, so that we have to consider, for example, that the heat capacity associated to the air can be affected by the mass of the building furniture.

For the same structural reason, also other parameters deserve some clarifications.

Some layers, for example, compose the external wall, and it is not trivial to define the position of the mass centre based on the collected data. This can cause some shift in the values of G_{ap} and G_{pe} . What

can be evaluated is only the total conductance of the wall (G_{ap} composed with G_{pe}) because we only have the external temperature and the inner air temperature data without any information on the wall internal temperature.

Another important parameter to evaluate is α , which influence the heat exchange between the distribution system and the indoor air.

The result (value closer to 0 than to 1) confirms the expected behaviour of the underfloor system.

To prove the good result in parameter identification, it is also important to evaluate the time constants associated to the model. In Table 4, these time constants are listed.

Table 4 – Model time constants

Time constant	Value	Unit
tau1	1.8	h
tau2	4.5	h
tau3	40	h

The last time constant (tau3), an order of magnitude larger than the others, is associated to the walls dynamics and has a reasonable value considering the dimension of the building.

Considering what discussed above, and the results shown in Table 2, we can confirm that the identification result can be considered good enough to move to the second step: the validation of the results using a different data set.

4.2 Validation Results

Once the model is tuned, based on the tuning data set, it is crucial to confirm its validity in reproducing the main dynamics of the system in general conditions, i.e. under different inputs.

A simulation of the tuned model is performed on a different data set.

Validation data range from 14/03/2015 to 28/03/2015. It is not a huge period but what is worth noticing is a change in the control strategy: during the weekend, the heating system is switched-off.

This change is evident if we consider Fig. 4 and Fig. 5 showing simulation results (red lines) and data (blue lines) of the indoor air temperature and the return water temperature.

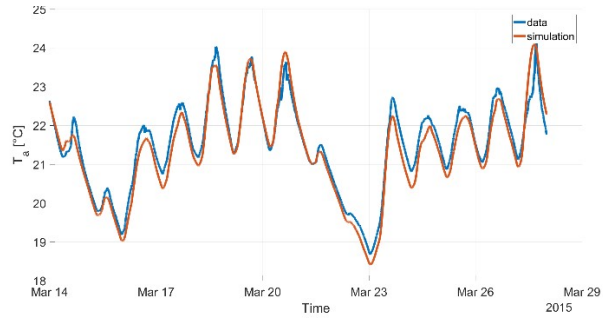


Fig. 4 – Simulation VS validation data: indoor air temperature

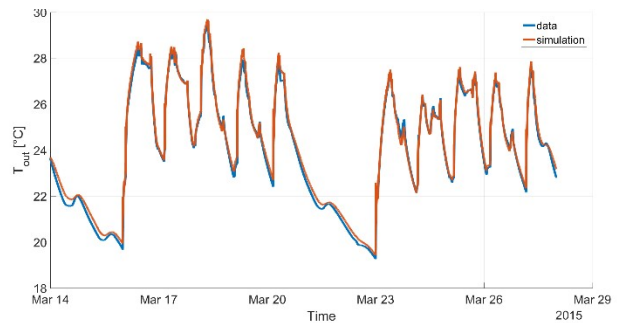


Fig. 5 – Simulation VS validation data: return water temperature

In Table 5, the same indexes used in section 4.1 are shown to evaluate the model deviation from the validation data set.

Table 5 – Model deviation from validation data

Index	T_a	T_{out}
RMSE	0.31	0.22
R^2	0.92	0.99

Considering Fig. 4, Fig. 5 and Table 5, we can confirm that the model can be used also under different conditions and the simulation results can be considered good enough for our purpose.

5. Conclusions

In the general framework of energy saving in buildings, the first step of a wider research on advanced control strategies is presented.

The choice of an appropriate model of the system is certainly a crucial part of the MPC approach chosen here.

Based on a case study, some important methodology aspects are treated.

First, the choice of the model structure and of input and output variables accordingly to the control purposes of the work is discussed.

Then the use of measured data during the normal operation mode of the system is presented and discussed as a good enough method to get data for the parameter identification process.

Unfortunately, the validation data set is not extensive enough to prove the validity of the tuned model on the entire winter period, but a future effort will be done to collect the desirable amount of data to carry out an extensive validation.

Moreover, a complete dissertation about the identification results in terms of physical meanings is here presented.

Nomenclature

Symbols

a	Solar coefficient (-)
b	Sharing coefficient for solar power (-)
C	Thermal capacity (J/K)
c	Water specific heat (J/kg/K)
G	Thermal conductance (W/K)
T	Temperature (°C)
\dot{m}	Water flow (kg/s)
α	Water temperature coefficient (-)
φ	Thermal flow (kW/m ²)
ϕ	Thermal power (kW)
o.r.	of reading

Subscripts/Superscripts

a	indoor air
ext/e	outdoor air
in	water flow
o	people and internal heat gains
out	return water flow
p	wall
r	solar global irradiance
s	floor

References

- ASHRAE. 2001. 2001 *ASHRAE Handbook-Fundamentals*. Atlanta, U.S.A.: American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc.
- Bacher, P., H. Madsen. 2011. "Identifying suitable models for the heat dynamics of buildings". *Energy and Buildings* 43(7): 1511-1522. doi: 10.1016/j.enbuild.2011.02.005.
- Bittanti, S. 2005. *Identificazione dei Modelli e Sistemi Adattativi*. Bologna, Italy: Pitagora Editrice.
- ISO. 1999. *ISO 9060:1999 - Solar energy - Specification and classification of instruments for measuring hemispherical solar and direct solar radiation*. Geneva, Switzerland: ISO.
- Kumara, R., R.K. Aggarwal, J.D. Sharma. 2013. "Energy analysis of a building using artificial neural network: A review". *Energy and Buildings* 65: 352-358. doi: 10.1016/j.enbuild.2013.06.007.
- Liao, Z., A.L. Dexter. 2004. "The potential for energy saving in heating systems through improving boiler controls". *Energy and Buildings* 36(3): 261-271. doi: 10.1016/j.enbuild.2003.12.006.
- Nocedal, J., S. Wright. 1999. *Numerical Optimization*. New York, U.S.A.: Springer.
- Parnis, G.A., A.B. Sproul. 2010. "Fast Thermal Modelling Using Micro-Cap". In: *Solar2010, the 48th AuSES Annual Conference*. Canberra, Australia.
- Parry, M., O. Canziani, J. Palutikof, P. van der Linden, C. Hanson. 2007. *Climate change 2007: impacts, adaptation and vulnerability*. Cambridge, UK: Cambridge University Press.
- Prívvara, S., Z. Váňa, E. Žáčková, J. Cigler. 2012. "Building modeling: Selection of the most appropriate model for predictive control". *Energy and Buildings* 55: 341-350. doi: 10.1016/j.enbuild.2012.08.040.