Analysis of Simplified Lumped-Capacitance Models to Simulate Thermal Behaviour of Buildings

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Abstract

Lumped-capacitance models for the simulation of the dynamic thermal behaviour of buildings have recently received growing attention due to their low computational cost and ease of implementation in city district simulation models. This work looks at two simplified dynamic models to evaluate the energy performance of buildings in both heating and cooling. In particular, the xRyC models proposed by the International Standard ISO 13790 and the German Guideline VDI 6007 are analysed in detail and compared with TRNSYS, in both long- and short-term, under the same boundary conditions. The analysis has been carried out considering an apartment with different types of building structures (high-low thermal capacitance, high-low thermal insulation). Four European climates (Helsinki, Venice, Vienna and Palermo) have been taken into account. The comparison has been done in terms of energy need, peak load, and hourly heating/cooling load profile during both seasons. The simulation results show that the simplified 7R2C model of the VDI 6007 is in good agreement with TRNSYS, in terms of both energy needs and transient behaviour. The improvement over the 5R1C model of the Standard EN 13790 increases when the cooling season is considered.

1. Introduction

Appropriate dynamic models are necessary both during early design stages of buildings and neighbourhoods as well as when retrofitting solutions for existing ones are being evaluated. The most frequently used software for building simulation - e.g. TRNSYS (Klein et al., 2010), EnergyPlus (Crawley et al., 2001) - rely on models that require a detailed description of the building in terms of both its

geometry and physical properties. As these methods require a high computational effort and cannot readily be interfaced with optimization solvers, they are unsuitable for applications such as model predictive control (Privara et al., 2013) and simulations of neighbourhoods or city districts (Kämpf et al., 2007). Secondly, due to the high number of input parameters they can hardly be approached by inexpert users. Thus, simplified dynamic models of buildings have been receiving growing attention in recent times. In these models, the system domain is discretized into a set of nodes connected by thermal resistances and capacitances (i.e. the parameters of the model). These parameters can be identified analytically, by model order reduction (Gouda et al., 2002) or by tuning the model to a temperature and energy consumption dataset (Madsen and Holst, 1995). The first attempt to describe the dynamic behaviour of building elements using the electrical analogy dates back to 1936 when Beuken (1936) derived the equations for a generic network with n thermal capacitances. That study formed the basis for the n-capacitances room model proposed by Rouvel (1972) that was implemented in the GEBSIMU software (Rouvel, 2015). A few years later, Laret (1980) proposed a simple analytical method to represent each construction element with a one-node model consisting of two resistances and one capacitance. The subdivision of the thermal resistance was carried out by calculating a variable referred to as the 'accessibility factor'. Based on this approach, Lorenz and Masy (1982) lumped all the construction elements of the thermal zone considered in a simple model with two time constants: one for the air volume and one for the building structure. In accordance with the same approach used for the n-capacitances model (Rouvel, 1972; Rouvel and Zimmermann, 2004) the building elements are split into those under symmetric load and those under asymmetric load, which are typically internal partitions and external walls, respectively. The 7R2C model of Rouvel and Zimmermann has been recently implemented in the German Guideline VDI 6007-1 (German Association of Engineers, 2012). The International Standard ISO 13790 (ISO, 2008) fully prescribes a quasi-steadystate calculation method - monthly method - and a simple dynamic method - simple hourly method based on a lumped-capacitance model with five thermal resistances and one thermal capacitance (5R1C).

In this work, an apartment was simulated with the one-capacitance (1C) model described in ISO 13790 and with the two-capacitances (2C) model described in VDI 6007, by taking into consideration four diverse building structures under four different climate conditions (Palermo, Venice, Vienna, and Helsinki). The results of the simplified models were then compared with those obtained from the wellestablished TRNSYS software (used as benchmark), in both heating and cooling seasons in terms of seasonal energy needs, peak load, and transient thermal behaviour.

2. Methods

2.1 Weather Conditions

One-hour time steps were used over a one-year simulation period including a heating season (from October 15 to April 15), a cooling season (from May 15 to September 15); the remaining periods of the year had free-floating indoor air temperatures. The test reference year (TRY) files of four reference European locations were used to examine a wide range of weather conditions. Table 1 summarizes the main characteristics of the climates mentioned. Both heating and cooling degree-days are calculated with a baseline of 18.3 °C. Solar irradiation refers to the annual amount of global radiation on the horizontal surface. Table 1 - Characteristics of the considered reference climates

	PA	VE	WI	HE
Max. temp.	34.6	33.6	31.7	28.7
[°C]				
Min. temp.	5.9	-5.8	-18.3	-21.7
[°C]				
Heating	801	2267	3180	4856
degree days				
Cooling	1002	474	212	39
degree days				
Ann. solar irr.	1458	1102	1123	947
[kWh/m ²]				

2.2 Buildings Structures

Four building structures representing different combinations of weight and thermal insulations [heavyweight not insulated (H1), heavyweight insulated (H2), lightweight not insulated (L1) and lightweight insulated (L2)] were considered. The building components are outlined in Table 2.

Table 2 - Characteristics of the building components

Building		U	m_{f}
components		$[W/m^2K]$	[kg/m ²]
External	EW_H1	1.06	410
walls	EW_H2	0.26	410
	EW_L1	1.04	50
	EW_L2	0.28	70
Internal	IW_H	2.53	150
partitions/	BO_H	0.95	310
boundary	IW_L	1.59	30
walls	BO_L	0.95	50
Ceilings/	CE_H	0.73	590
floors	FL_H	0.73	590
	CE_L	0.33	290
	FL_L	0.33	290
Windows	SP	5.68	-
	DP	2.83	-

Both simplified models used the solar heat gain obtained from the detailed simulation as the input signal. This was done because many authors have indicated that solar gains are one of the main sources of uncertainty (Reynders et al., 2014). Solar gains were calculated using Type 56 of TRNSYS. At each time step, the solar heat entering the building was given by the sum of shortwave transmission and secondary heat flux through external windows. The apparent sky temperature and the ground temperature were calculated according to the equations proposed by the Standard VDI 6007. Table 3 outlines the combination of building components of each building structure.

Table 3 – Building components of the reference envelopes

Envelope \	Non-	Adiabatic
building comp.	Adiabatic	
Heavyweight	EW_H1	IW_H, BO_H,
uninsulated (H1)	SP	CE_H, FL_H
Heavyweight well	EW_H2	IW_H, BO_H,
insulated (H2)	DP	CE_H, FL_H
Lightweight	EW_L1	IW_L, BO_L,
uninsulated (L1)	SP	CE_L, FL_L
Lightweight	EW_L2	IW_L, BO_L,
well insulated (L2)	DP	CE_L, FL_L

Other boundary conditions for both simplified and detailed building models are: emissivity of surfaces for long-wave thermal radiation (0.9); absorption coefficient of exterior surfaces (0.6); convection heat transfer coefficient of internal surfaces (2.7 W/(m² K) for horizontal heat flow and 1.7 W/(m² K) for vertical heat flow) and convection heat transfer coefficient of external surfaces (20 W/(m² K)).



Fig. 1 - Floor plan of the apartment

The considered building was a 93.6 m² single-storey apartment (see Fig. 1). The apartment has two external walls oriented along the east-west direction - or north-south, depending on the considered simulation - and the construction elements in all the other directions were considered adiabatic. The latter include internal partitions, boundary walls towards conditioned spaces (adjacent apartments), floor and ceiling. All glazed components on the east side are windows of height 1.3 m (overall glazed area of 5 m²), whereas all glazed components on the opposite side are doors of height 2.3 m (overall glazed area of 9.4 m²). The height of the apartment was 3 m and the overall surface of the internal partitions was 95 m². Both the air mass flow rate due to infiltration and/or natural ventilation and the heat flow rate due to internal heat gains were assumed constant in time and uniform within the thermal zone.

They were calculated in accordance with the Italian Standard UNI/TS 11300-1 (UNI, 2008). The internal heat gain was calculated according to Eq. (1) and the air change rate was assumed to be 0.5 volumes per hour, as is generally assumed for domestic dwellings.

$$\Phi_{\rm int} = 5.294 \ A_{\rm fl} - 0.01557 \ A_{\rm fl}^{2} \ [W] \tag{1}$$

For sake of simplicity, this paper includes only the numerical results of the apartment oriented along the east-west axis. Analogue results were obtained for the north-south case.

2.3 Evaluation of Models Accuracy

The different load profiles obtained by the RC models were compared to assess their accuracy with respect to the TRNSYS model as far as the dynamic response and overall energy needs were concerned. The dynamic response was evaluated by calculating the distance between the heat load profiles of the RC models and the heat load profile obtained with TRNSYS, i.e. the root mean squared error (RMSE). In order to compare the RMSE obtained with reference to different building structures and under different weather conditions, the latter was successively normalized with respect to the mean heat load of the case considered, thus obtaining the relative error ε . The mean heating/cooling load was obtained by dividing the energy needed for space heating/cooling by the number of hours with HVAC systems turned on.

Simplified Models

Lumped-capacitance models assume that the distributed thermal mass of the dwelling is lumped into a discrete number of thermal capacitances, depending on the model type (Reynders et al., 2014). The lumped-capacitance model was solved by a linear system composed of n heat balance equations, where n is the number of nodes of the corresponding thermal network. As is usual in building simulations, the system has one degree of freedom unless one variable is fixed by the user. This leads to two possible model uses:

Calculation of the heat load: The indoor air temperature $\theta_i = \theta_{set}$ is set by the user and the output of the model is the heat load ϕ_{hc} ;

Calculation of the indoor air temperature: The user sets the heat load ϕ_{hc} and the output of the model is the indoor air temperature θ_i .

The models considered do not include the balance of water vapour in the indoor ambient, which means that the calculation of the latent heat load (to be delivered to or extracted from conditioned spaces) is not included. The models were developed in the MATLAB environment (Mathworks, 2010); their main features are briefly described in the following.

3.1 The 5R1C Model



Fig. 2 - 5R1C model of EN ISO 13790

The International Standard ISO 13790 presents two methods that rely on the same inputs to calculate the building's energy use at different levels of detail: the *monthly method* with one month time intervals and the *simple hourly method* with one hour time intervals. The simple hourly method is based on the equivalent resistance-capacitance (RC) circuit shown in Fig. 2.

3.2 The 7R2C Model



Fig. 3 - 7R2C model of VDI 6007

The model distinguishes between adiabatic and non-adiabatic building components and assigns the thermal capacity to each of the two groups. In the 5R1C model, the entire thermal mass of the building is lumped into one single element, thereby making this distinction impossible. Both models use the ISO 13786 Standard (ISO, 2007) for the calculation of the thermal capacitances. The equivalent circuit is shown in Fig. 3.

4. Results

4.1 Peak Loads and Energy Needs

Table 4 shows that lumped-capacitance models tend to slightly underestimate the peak load for the space heating of uninsulated building structures (down to -5.5 % with the 1C model and to -2.2 % with the 2C model) and to overestimate it for well insulated buildings (up to +8.1 % with the 1C model and to +4.0 % with the 2C model).

Tables 6 and 7 seem to extend the patterns found for the peak load to the energy needs calculation. In fact, the energy needs for space heating is slightly underestimated by the RC models for buildings with low thermal insulation (approx. -4 % for both models), while it is overestimated for highly insulated building envelopes (+5 % for the 1C model and +3 % for the 2C model). The trends in the space cooling mode remained almost entirely unaltered, although in this case the error in the lumped-capacitance models was not as significant as that in the peak load calculation: the mean error of the simulations with the 1C model is approx. -2.2 % and drops to -0.3 % with the 2C model. This fact suggests that although they do not accurately follow the fluctuations of the heat load, simplified models are better able to estimate the energy needs season. On the other hand, the peak load for space cooling is systematically underestimated (from -6.3 % to -14.5 %) by the 1C model, as it can be observed in Table 5. The 2C model seems instead to be quite accurate, with an error that ranges from -2.7 to +4.1 % and a mean error of +0.3 %.

Env.	Clim.	Peak load		
		for space heating [W]		
		TRNSYS	5R1C	7R2C
	PA	1564	-0.5 %	-2.2 %
H1	VE	3321	-3.6 %	-1.7 %
	WI	4882	-1.2 %	-0.3 %
	HE	6037	-2.6 %	-1.7 %
	PA	717	+8.1 %	+4.0 %
H2	VE	1745	+2.0 %	+2.5 %
	WI	2629	+2.5 %	+3.4 %
	HE	3277	+2.0 %	+2.0 %
	PA	1761	-4.8 %	+0.1 %
L1	VE	3560	-5.5 %	-0.7 %
	WI	5355	-4.6 %	-0.9 %
	HE	6223	-3.8 %	-1.5 %
	PA	803	+6.6 %	+2.9 %
L2	VE	1831	+2.9 %	+2.0 %
	WI	2796	+2.7 %	+3.4 %
	HE	3378	+1.7 %	+1.7 %

Table 4 – Peak load for space heating

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Env.	Clim.	Energy needs for space heating [kWh]		
		TRNSYS	5R1C	7R2C
	PA	1293	-2.5 %	-6.5 %
H1	VE	7377	-4.0 %	-4.2 %
	WI	9304	-4.3 %	-4.0 %
	HE	13557	-4.2 %	-3.6 %
	PA	137	+33 %	+16 %
H2	VE	3055	+5.7 %	+3.4 %
	WI	3890	+5.7 %	+3.8 %
	HE	6528	+3.7 %	+2.4 %
	PA	1336	-5.2 %	-6.3 %
L1	VE	7300	-3.5 %	-3.5 %
	WI	9203	-3.8 %	-3.3 %
	HE	13291	-3.3 %	-2.9 %
	PA	179	+14 %	+7.3 %
L2	VE	3129	+4.8 %	+2.7 %
	WI	3989	+4.6 %	+3.2 %
	HE	6531	+4.0 %	+2.5 %

Table 6 – Seasonal energy needs for space heating

Table 5 – Peak load for space cooling

Env.	Clim.	Peak load		
		for space cooling [W]		
		TRNSYS	5R1C	7R2C
	PA	2720	-10.5 %	-1.5 %
H1	VE	2678	-6.3 %	-0.8 %
	WI	3810	-7.7 %	+0.4 %
	HE	2582	-5.7 %	+2.5 %
	PA	2346	-11.1 %	-1.2 %
H2	VE	2307	-8.1 %	-0.4 %
	WI	3390	-9.9 %	+1.4 %
	HE	2571	-9.1 %	+1.8 %
	PA	3164	-14.5 %	+1.3 %
L1	VE	3184	-12.9 %	+1.7 %
	WI	4602	-13.8 %	+2.0 %
	HE	3290	-13.1 %	+4.1 %
	PA	2637	-13.8 %	-2.7 %
L2	VE	2572	-10.8 %	-2.1 %
	WI	3895	-13.4 %	-0.8 %
	HE	3003	-12.6 %	-0.4 %

Table 7 – Seasonal	energy needs	for space cooling
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Env.	Clim.	Energy needs		
		for space cooling [kWh]		
		TRNSYS	5R1C	7R2C
	PA	3353	-3.6 %	-1.3 %
	VE	2170	-3.0 %	-0.4 %
H1	WI	2282	-0.1 %	0.0 %
	HE	809	+4.2 %	+3.3 %
	PA	3402	-2.9 %	-1.3 %
H2	VE	2589	-3.0 %	-1.0 %
	WI	3071	-3.2 %	-1.9 %
	HE	1811	-5.1 %	-2.8 %
	PA	3313	-2.4 %	+0.1 %
	VE	2228	-3.5 %	+1.2 %
L1	WI	2409	-2.1 %	+1.7 %
	HE	952	-3.7 %	+4.7 %
	PA	3337	-2.9 %	-1.4 %
L2	VE	2578	-3.2 %	-1.1 %
	WI	3017	-2.9 %	-1.5 %
	HE	1794	-5.1 %	-2.5 %

4.2 Transient Behaviour

The accuracy in the transient response of the lumped-capacitance models is measured here as the mean distance (RMSE) between their heating/cooling load profiles and those obtained by TRNSYS simulations. Fig. 4 shows the RMSE of the simplified models and the peak load in the corresponding season for the apartment facing east west. The peak load, which was extracted from the heat load profile produced by TRNSYS, serves as a valid reference for both simplified models. The RMSE of the simplified models in heating mode increased from Palermo to Vienna but showed a smaller increase with reference to the snow-dominated climate of Helsinki. Fig. 4(a), for example, shows that while the peak load in Vienna is more than three times the peak load in Palermo (from 1564 W to 4882 W) and the $RMSE_{1C}$ is more than twice (from 81 W to 184 W), this proportion is no longer valid when we turn our attention from Vienna to Helsinki (+23 % of peak load and only +5 % of the RMSE_{1C}). This trend could be linked to the different patterns of temperature differences between the indoors and outdoors, which represents the main driving force for space heating load in winter months. Indeed, temperatures are less prone to register significant diurnal fluctuations in the Nordic climate of Helsinki than in the other locations considered in this study. This trend seems to be valid for both simplified models regardless of the building envelope. During the warm season, instead, the cooling load is a result of the overlapping effects of outdoor air temperature and solar radiation. Fig. 4 shows that the accuracy of the 7R2C model is greater than that of the 5R1C one in all the cases considered and that such improvement is particularly relevant in the cooling season. Indeed, the RMSE from 1C to 2C drops from -35 % to -53 % in heating mode and from -44 % to -76 % in cooling mode.



Fig. 4 – RMSE of lumped-capacitance models and peak load for the apartment with envelopes: (a) H1, (b) H2, (c) L1 and (d) L2

The trivial reason for this improvement is linked to the presence of the second thermal capacitance that makes it possible to distinguish between adiabatic and asymmetrically loaded building components, as explained in Section 3.2. Moreover, the presence of two internal surface temperature nodes (θ_{IW} and θ_{AW} in Fig. 3) may lead to further improvement with respect to the 1C model due to both a more coherent distribution of heat gains throughout the wall surfaces and the introduction of the radiative heat exchange between the inside surface of external walls and the surfaces of internal building components.

The relative errors ε of both simplified models are presented graphically in Fig. 5 for the 32 simulations of the apartment (with both orientations). Here, it is evident that the type of building structure does not significantly affect the accuracy of the simplified models in the heating mode, as the effect of climate conditions prevails. In fact, the blue indicators that represent the climate of Helsinki are always lower than the red ones that represent the climate of Palermo, while the green and the orange indicators are always somewhere in between. This holds true both for the 1C and 2C models, although the error of the former presents a higher dispersion. The improvement of the 2C model over the 1C one is evident for both seasons. Fig. 5 shows that while the error \mathcal{E}_{1C} of the 1C model shoots to very high values (20-30 %) when the transition is made from the heating to cooling mode, the error \mathcal{E}_{2C} of the 2C model does not undergo such a sharp increase and always remains below the threshold of 12.5 %.



Fig. 5 – The relative error $\boldsymbol{\epsilon}$ in the heating and cooling seasons

Contrary to what takes place in the heating mode, the type of building envelope does seem to affect the accuracy of the models during the warm season. Indeed, lightweight building structures present a higher error than heavyweight ones. This is particularly evident when the error of the 2C model is compared, since \mathcal{E}_{2C} increases from 6–7 % to 12 % when there is low thermal insulation (i.e. from H1 to L1) and from 7–11 % to 10–12 % when it is high (i.e. from H2 to L2). This may be due to the difficulty that lumped capacitance models have in following the reaction of lightweight structures to rapid fluctuations of air temperature or heat gains.

5. Conclusions

The 5R1C and 7R2C lumped-capacitance models described in the International Standard ISO 13790 and in the German Guideline VDI 6007 were used to simulate the thermal behaviour of an apartment using four reference building envelopes in four different climates. The accuracy was evaluated by comparing the resulting profiles with those obtained by the well-established software TRNSYS.

Both lumped-capacitance models appear to reliably calculate the overall energy needs of buildings in both heating and cooling seasons.

As far as the transient behaviour is concerned, the second-order model of VDI 6007 provides more accurate results. Indeed, the first-order model systematically underestimated the peak load for space cooling (-11 % on average), while the second-order model showed a fairly accurate calculation. The accuracy of the 7R2C model in terms of relative error ε (ratio between the RMSE and the mean load) was approximately 6 % and 9 % in the heating and cooling seasons, respectively.

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Nomenclature

Symbols

Α	Surface area (m ²)
С	Thermal capacitance (J/K)
Н	Heat transfer coefficient (W/K)
m_f	Frontal mass (kg/m ²)
R	Thermal resistance (K/W)
RMSE	Root mean squared error
U	Thermal transmittance (W/(m ² K))
ε	Relative error (-)
θ	Temperature (K)
ϕ	Heat flow rate (W)

Subscripts/Superscripts

AW	Non-adiabatic building elements
conv	Convective
е	External air
fl	Floor
hc	Heating/cooling load
i	Indoor air
int	Internal gains
IW	Adiabatic building elements
т	Thermal mass (node)
rad	Radiative
S	Surface (node)
set	Set-point
sup	Supply (air)
tr	Transmission
ve	Ventilation

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