Stochastic Multi-Criteria Decision Making of Energy Recovery Ventilation Systems using Cumulative Prospect Theory

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Abstract

Recently, an Energy Recovery Ventilator (ERV) in a residential building was seen as an attractive ventilation option in terms of energy saving and indoor air quality. In order to identify a feasible set among many ventilation strategies in this situation, various decision-making approaches (deterministic or stochastic) using Building Performance Simulation (BPS) tools have been suggested. As a simulation-based decision-making approach, a Stochastic Multi-Criteria Decision Making (SMCDM) method based on Cumulative Prospect Theory (CPT) is presented in this paper to find the best ventilation strategy under model uncertainties. For this study, two ventilation strategies, considering air inlet positions and CO_2 sensor positions, were chosen and modelled using two simulation tools: CONTAMW 3.1 for the airflow model and EnergyPlus for the thermal model. In addition, Latin Hypercube Sampling (LHS) was used to reflect the model uncertainties. In this study, it is shown that CPT can provide a more realistic and trustworthy framework than the Bayesian decision theory.

1. Introduction

Due to the high attention given to passive houses and the increase in toxic air environments, an Energy Recovery Ventilator (ERV) in a residential building is being installed in order to attain acceptable Indoor Air Quality (IAQ) and to reduce energy consumption. The ERV is critical for people who spend about 90 % of the day in indoor spaces (Laverge and Janssens, 2013), and a ventilation strategy decision-making is emerging as a major issue.

Building Performance Simulation (BPS) tools can obtain predicted outputs (energy consumption, thermal comfort, CO_2 concentration, etc.) through a kernel engine in a mathematical model, considering the indoor and outdoor physical environmental conditions. Such predicted outputs can be used to determine the optimal design of the ERV. However, the BPS tools have many unknown inputs that generate uncertainty of the predicted outputs. Furthermore, such uncertainty is a major issue in finding a highly reliable design alternative (de Wit, 2001; Macdonald, 2002; Hopfe, 2009; Kim et al., 2014; Sun et al., 2014).

Previous studies (de Wit, 2001; Kim et al., 2014) have suggested the Monte Carlo Simulation (MCS) and the Multi-Criteria Decision Making (MCDM) under uncertainties using the Bayesian decision theory to deal with the stochastic decision-making issues. They showed the differences between the deterministic and stochastic approach, as well as the possibility of reaching a meaningful decision-making result. The Bayesian decision theory is used to calculate the utility function reflecting the preferences or attitudes of decision makers toward risk, and determine a design alternative with high-expected utility. However, the Bayesian decision theory based on utility function is problematic since the decision-making problem is solved under the assumption that the decision makers behave rationally (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992). Because the individual cognitive ability varies among general decision makers, it is difficult to assume they are rational participants. To handle the aforementioned issue, Cumulative Prospect Theory (CPT) has been presented (Lahdelma and Salminen, 2009; Wakker, 2010; Krohling and de Souza, 2012).

In this study, to solve a Stochastic Multi-Criteria Decision Making (SMCDM) problem, the CPT is developed as an alternative to utility function and is used to identify a feasible set among many ventilation strategies of the ERV in a given residential building.

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2. Stochastic Multi-Criteria Decision Making (SMCDM)

Most decision-making problems using BPS tools are based on many criteria rather than a single criterion. The optimal alternative is identified using predicted outputs. It should be noticed that the predicted outputs are probabilistic rather than deterministic, due to various uncertainty sources (aleatory or epistemic uncertainties). In other words, decision-making problems using BPS tools are Stochastic Multi-Criteria Decision Making (SMCDM) that must reflect various risks under uncertainties and multi-criteria simultaneously.

The SMCDM methods can be used as follows: (1) Bayesian decision theory, (2) Cumulative Prospect Theory (CPT). The Bayesian decision theory can reflect the preferences for risks under uncertainty and determine an alternative with high-expected utility (Von Neumann & Morgenstern, 1947). To present the decision-making process with the Bayesian decision theory in the area of building simulation, de Wit (2001) selected initial cost and thermal comfort as multi-criteria problems and treated a MCDM problem using stochastic predicted outputs propagated by MCS and the joint utility function. Kim et al. (2014) showed the feasibility of Bayesian inference based on the Markov Chain Monte Carlo (MCMC) to consider the different expected utilities of multiple decision makers, rather than of only a single decision-maker. The aforementioned previous studies are signifycant in terms of making reference to which multi-criteria decision-making was conducted by reflecting the risks of stochastic predicted outputs.

However, the Bayesian decision theory is problematic since it assumes that decision makers rationally recognize utility and behave with a consideration of the risks (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992). It is difficult for decision makers to find a highly reliable alternative, considering they are based on vague information or data, and each decision maker has different cognitive abilities. In contrast, the CPT can resolve the decision-making problem based on the utility theory by reflecting reference point setting, diminishing sensitivity, and loss aversion (Lahdelma and Salminen, 2009; Wakker, 2010; Krohling and de Souza, 2012).

- Reference point setting: when making a valueble decision about gains and losses, decision makers decide a value relatively, rather than absolutely, by comparing against the predefined individual reference point. This relative valuation differs considerably from the utility theory based on the absolute value. Since decision makers' preferences toward gains and losses differ based on the reference point, difference value functions must be applied. The CPT can distinguish gains and losses according to the reference point setting and express them as value functions having an asymmetrical s-shape, as shown in Fig. 1.
- Diminishing sensitivity: even if two design alternatives have the same difference in gain or loss, they have a large value change if the difference between gain or loss and the reference point is small. Otherwise, they have a small value change. It is called diminishing sensitivity. As shown in Fig. 1, the slope of the value function decreases if the difference between gain or loss and the reference point increases. The CPT can reflect the diminishing sensitivity of the decision makers by varying the weighting function of gain or loss based on the reference point.
- Loss aversion: decision makers tend to show higher loss aversion for losses than for gains according to previous studies (Wakker, 2010; Krohling and de Souza, 2012). In other words, decision makers are more sensitive to losses than to gains. The CPT can distinguish the value functions of gains and losses according to the reference point and reflect a loss aversion coefficient for losses.
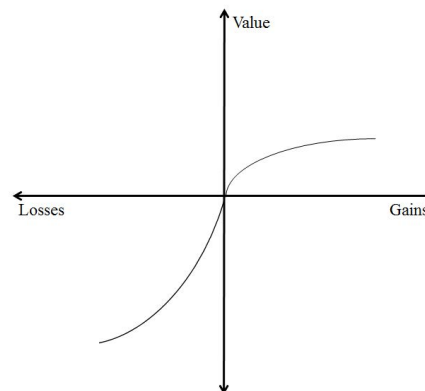

Fig. 1 - Value function of CPT

Within the aforementioned mind, this study uses the CPT for SMCDM. The prospect value $V(f)$ of CPT can be expressed as Equation (1) using the value function $v(x)$, the decision weight function π, and the probability of event p. The value function is a function according to gains if the difference between criteria value (predicted output) and reference point is positive, or a function according to losses if the difference is negative (Equation (2)). The parameters α, β related gains and losses capture the concave and convex curvature of the value function (Krohling and de Souza, 2012), and λ is the loss aversion coefficient that is used to reflect a high loss aversion toward loss. Kahneman and Tversky (1979) proposed α =0.88, β =0.88, λ =2.25, and these parameters were used in this study. The weighting function reflects the diminishing sensitivity using the attitude coefficients γ, ϕ of gains and losses for risk as shown in Equations (3)-(6). The attitude coefficient of gains and losses were set as $\gamma^{+,-}$ =0.8 and ϕ =1.0, respectively, as suggested by Prelec (1998). The propagated stochastic predictions (heating energy and CO2 concentration) using the MCS were used in this study to calculate the probability of an event.

$$V(f) = V(f^-) + V(f^+) = \sum_{i=1}^{h} \pi_i^- v(x_i) + \sum_{i=h+1}^{n} \pi_i^+ v(x_i) \quad (1)$$

$$v(x) = \begin{cases} x^\alpha & x \geq 0 \\ -\lambda(-x)^\beta & x < 0 \end{cases} \quad (2)$$

$$\pi_i^+ = \omega^+\left(\sum_{j=i}^{n} p_j\right) - \omega^+\left(\sum_{j=i+1}^{n} p_j\right) \quad (3)$$

$$\pi_i^- = \omega^-\left(\sum_{j=1}^{i} p_j\right) - \omega^-\left(\sum_{j=1}^{i-1} p_j\right) \quad (4)$$

$$\omega^+\left(\sum_{j=h}^{n} p_j\right) = \exp\left(-\gamma^+\left(-\ln\left(\sum_{j=h}^{n} p_j\right)\right)^\phi\right) \quad (5)$$

$$\omega^-\left(\sum_{j=1}^{h} p_j\right) = \exp\left(-\gamma^-\left(-\ln\left(\sum_{j=1}^{h} p_j\right)\right)^\phi\right) \quad (6)$$

where, $V(f)$ is the prospect value, $v(x)$ is the value function, x is the difference between the critical value and the reference point, α, β are the parameters related to gains and losses, λ is the loss aversion coefficient, π is the decision weight function (+ and – superscript denote gain and loss, respectively.), ω is the weighting function, γ is the risk gain attitude coefficient, and ϕ is the risk loss attitude coefficient.

3. Target Building and Unknown Inputs

3.1 BPS Tools

For the target building of this study, a 15-story residential building in Seoul, Korea was selected for the analysis of the ventilation strategies (Fig. 2(a)). To simulate thermal and airflow phenomena, CONTAMW 3.1 and EnergyPlus 8.0 were chosen (Fig. 2(b)-(c)). EnergyPlus has been used extensively to calculate transient heat and mass flow. But it cannot perform duct modelling. Otherwise, CONTAMW 3.1, adequate for determining macro flow phenomena such as overall ventilation rates, enables the duct modeling, although it cannot reflect dynamic energy flows such as indoor air temperature. To solve these problems, the present study integrated two BPS tools using a Ping-Pong method (decoupled approach).

The simulation period was one day in winter (January 21), and the Seoul climate data was used. It was assumed that all doors of the room were open and the windows were closed. Since most occupants do not actively open windows in winter and this is appropriate to assess the IAQ (Kim and Park, 2009). An adult was assumed to generate 0.31 liters per minute. Infiltration was taken into consideration. The occupant schedules employed the data provided by Hyun and Park. (2006).
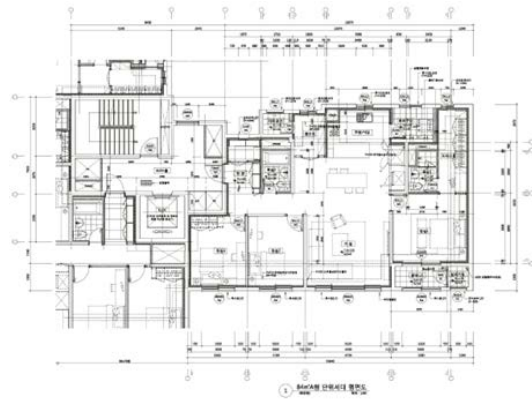
(a) Floor plan
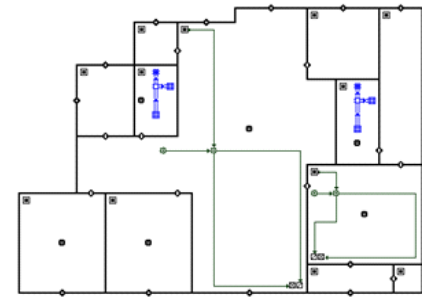
(b) CONTAMW 3.1
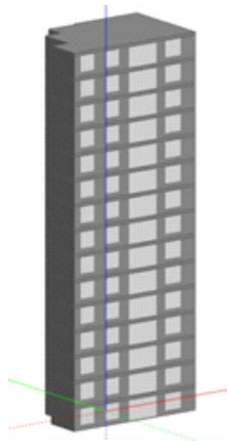
(c) DesignBuilder

Fig. 2 - Target residential building and BPS tools

Fig. 3 shows the average occupant schedules. And the same occupant schedule for the adjacent rooms (master room, living room) was used for the bathroom. The radiant floor heating system was controlled per room and on/off control method was applied based on the heating set-point temperature (20 ℃). For the ERV, the CO_2-sensor based Demand-Controlled Ventilation (DCV-CO_2) was selected. The DCV-CO_2 is operated using the on/off control method based 1,000 PPM. The air supply and exhaust rate were set as 100 CMH
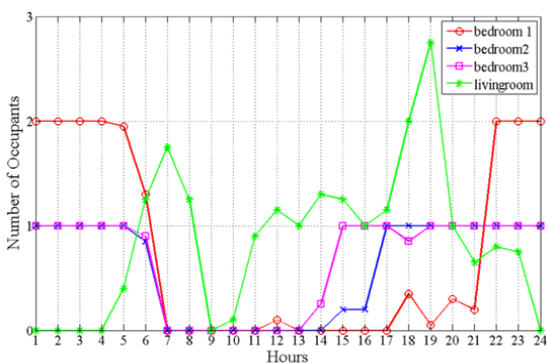

Fig. 3 - Occupant schedule (Hyun and Park, 2006)

(Cubic Meter per Hour) and 60 CMH for the air exhaust system in the bathroom.

As mentioned above, the Ping-Pong method was used to complement the shortcomings of the two simulation models. The EnergyPlus model was applied to the radiant floor heating system and ERV, and CONTAMW 3.1 was only applied to ERV. And each system is operated by automatic control logic (on/off control for radiant floor heating system, DCV-CO_2 control for ERV). For the Ping-Pong approach, the two simulation models were integrated in MATLAB platform. For each time step, EnergyPlus calculates indoor air temperatures and then CONTAMW 3.1 recalculates the airflows of the openings (windows and doors) and/or ducts, air supply rate of ERV, and CO_2 concentration using the calculated indoor air temperatures. The calculated airflows were automatically input to Energy-Plus and the heating energy consumption was recalculated. The aforementioned coupling process was repeated every 20 minutes.

3.2 Selection of Unknown Inputs

The unknown inputs of the BPS tools were chosen as shown in Table 1 by referring to the previous studies (Hyun and Park, 2006; ASHRAE, 2013; DOE, 2013a; DOE 2013b, Kim et al., 2014; Macdonald, 2002; Hopfe, 2009; Walton & Dols, 2005). The unknown inputs were assumed to have a triangular distribution (T[0.9, 1.0, 1.1]) consisting of the minimum, maximum, and base values. The triangular distribution is propagated as ratio of the definite values of the unknown inputs. The occupant schedule was chosen as the discrete uniform distribution (D[1, 30]).

Table 1 – Unknown inputs

	Descriptions
Construction materials	Density, specific heat, and conductivity of gypsum board
	Density, specific heat, and conductivity of brick
	Density, specific heat, and conductivity of concrete
	Density, specific heat, and conductivity of insulation board
	Density, specific heat, and conductivity of acoustic tile

	Solar transmittance, reflectance, emissivity, conductivity of clear window
Numerical algorithm	Loads or temperature convergence tolerance value
Grounds	Temperature and reflectance
Set-point temperature	Heating set-point temperature
Internal heat gains	Number of person, activity level, fraction radiant of people (master room)
	Number of person, activity level, fraction radiant of people (bedroom1)
	Number of person, activity level, fraction radiant of people (bedroom2)
	Number of person, activity level, fraction radiant of people (living room)
	Number of person, activity level, fraction radiant of people (bathroom1)
	Number of person, activity level, fraction radiant of people (bathroom2)
	Internal gains and fraction radiant of lights
	Internal gains and fraction radiant of electric equipment
Schedule	Occupants' schedules
ERV	Fan efficiency, pressure rise, and motor efficiency of supply fan
	Fan efficiency, pressure rise, and motor efficiency of return fan
	Sensible or latent effectiveness
Exhaust fan	Fan efficiency and pressure rise of exhaust fan (bathroom1)
	Fan efficiency and pressure rise of exhaust fan (bathroom2)
Pumps	Rated pump head and motor efficiency of heating water circulation pumps
Plants	Maximum or minimum loop temperature
Airflows	Flow exponent, discharge coefficient, wind pressure coefficient, wind velocity profile exponent, local terrain constant, terminal loss coefficient, leakage class#1(oval), leakage class#2(rectangular), duct roughness, leakage area of doors, leakage area of windows

4. Uncertainty Results

For the propagation of uncertainties, Latin Hyper-cube Sampling (LHS), appropriate for complex non-linear models, was used. The number of sampling case was set to 200. The heating energy consumption is the sum total of the radiant floor heating system and ERV. The CO_2 concentration was expressed as CO_2 performance φ using the total occupation time of each room T and total time δ when CO_2 concentration is below 1,000 PPM as shown in Equation (7). In other words, the uncertainty results are represented as total heating energy consumption (*kWh*) and CO_2 performance (%). And the goal of this study was to determine the air inlet position of the ERV and CO_2 sensor positions. In other words, it is a SMCDM problem. For this study, two ventilation strategies were used as shown in Table 2.

$$\varphi = \left(\sum_{k=1}^{m} \frac{\delta_k}{T_k} \times 100 \right) / m \qquad (7)$$

where, φ is the CO_2 performance (%), δ is the total time when the CO_2 concentration of each room is below 1,000PPM (hour), T is the total occupation time of each room (hour), and m is the number of rooms (master room, bedroom 1, bedroom 2, and living room).

Table 2 – Two ventilation strategies according to outdoor air supply rate, air inlet position, and CO_2 sensor position

ALT.	Outdoor air supply rate	Air inlet position	CO2 sensor position
1	DCV-CO2	Living room	Living room
2		Living room + Master room	Living room + Master room

Table 3 shows the uncertainty results of two design alternatives. In terms of total energy consumption, ALT #1 is superior by a difference of 1.37 (*kWh*), but the difference is insignificant. In terms of CO_2 performance (%), ALT #2 is superior owing to the additional CO_2 sensors in the master room. In the results of the coefficient of variation, which expresses the degree of uncertainty, ALT #2 is superior in terms of total heating energy consumption and CO_2 performance, but the difference in the degree of

uncertainty is insignificant. Fig. 4 shows the un-certainty results of each alternative using the Probability Distribution Function (PDF).

Table 3 – Uncertainty results (STDEV: Standard deviation, COV: Coefficient of Variation)

	ALT #1		ALT #2	
	Heating energy (kWh)	CO2 performance (%)	Heating energy (kWh)	CO2 performance (%)
Min	40.86	12.61	42.23	29.27
Max	99.07	85.70	100.44	135.72
Mean	84.16	34.39	85.53	66.14
STDEV	11.25	13.17	11.26	19.49
COV	0.134	0.383	0.132	0.295
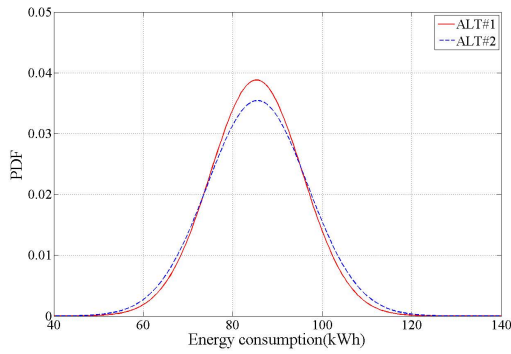
(a) Total heating energy consumption (kWh)
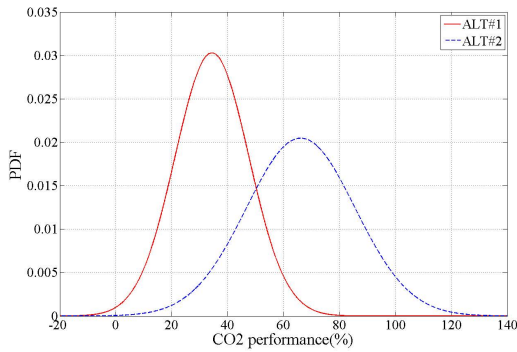
(b) CO2 performance (%)

Fig. 4 - Uncertainty results using PDF (ALT. #1 vs. ALT. #2)

5. SMCDM Results

In this study, reference points and weighting factors were selected to calculate the prospect values (Tables 4-5). The weighting factors were used to transfer into a single cost function. However, it should be noticed that the selection of reference points and weighting factors can be changed ac-cording to the preferences of the real decision makers.

Table 4 – What-if scenarios of reference points

Case	Heating energy (kWh)	CO2 performance (%)
1	70	20
2	70	25
3	75	20
4	75	25

Table 5 – What-if scenarios of weighting factors for multi-criteria

Weight	Heating energy (kWh)	CO2 performance (%)
1	0.8	0.2
2	0.2	0.8
3	0.5	0.5

Table 6 shows the prospect values of the CPT according to the scenarios of reference points and weighting factors. The prospect value is expressed as a normalized prospect value matrix (Table 7) using Equations (8) and (9). As shown in Table 8, the total prospect value reflecting the two performance criteria (heating energy consumption and CO2 performance) can be calculated using Equation (10).

$$r_{i,j} = \frac{v_{i,j}}{\max(v_{i,j})}, \qquad i \in M, j \in N_1 \tag{8}$$

$$r_{i,j} = \frac{\min(v_{i,j})}{v_{i,j}}, \qquad i \in M, j \in N_2 \tag{9}$$

$$v(\alpha_i) = \prod_{j=1}^{n} r(x_j)^{w(j)} \tag{10}$$

where, r is the normalized prospect value, v is the prospect value of each alternative (refer to Table 6), N_1 is the benefit criteria for CO2 performance, and N_2 is the cost criteria for the heating energy con-sumption.

Table 6 – Prospect values of alternatives

ALT	Case #1		Case #2		Case #3		Case #4	
	Heating energy	CO_2 performance	Heating energy	CO_2 performance	Heating energy	CO_2 performance	Heating energy	CO_2 performance
1	67.81	80.93	67.81	49.50	21.58	56.27	21.58	30.62
2	79.06	204.54	79.06	170.13	34.37	203.59	34.37	170.13

Table 7 – Normalized prospect matrix results

ALT	Case #1		Case #2		Case #3		Case #4	
	Heating energy	CO_2 performance	Heating energy	CO_2 performance	Heating energy	CO_2 performance	Heating energy	CO_2 performance
1	1	0.396	1	0.291	1	0.276	1	0.180
2	0.858	1	0.858	1	0.628	1	0.628	1

Table 8 – SMCDM results of ERV using CPT

ALT	Case #1			Case #2			Case #3			Case #4		
	Weight #1	Weight #2	Weight #3	Weight #1	Weight #2	Weight #3	Weight #1	Weight #2	Weight #3	Weight #1	Weight #2	Weight #3
1	0.83	0.48	0.63	0.83	0.37	0.54	**0.77**	0.36	0.53	**0.77**	0.25	0.42
2	**0.88**	**0.97**	**0.93**	**0.88**	**0.97**	**0.93**	0.69	**0.91**	**0.79**	0.69	**0.91**	**0.79**

Comparing with the total prospect values according to the reference point scenarios (Table 4) under the Weight #1 condition (Table 5), ALT #2 (total prospect value: 0.88) is the optimal alternative for Cases #1-2 and ALT #1 (total prospect value: 0.77) is the optimal alternative for Cases #3-4. When the weight scenario is changed (Weights #1-3 in Table 5), a different optimal alternative is determined for Cases #3-4 among the reference point scenarios. For Cases #1-2, ALT #2 is determined as the optimal alternative, but the total prospect value is changed depending on the weighting factor.

As shown in the above results, the SMCDM using CPT results in a different optimal alternative for the ERV depending on the reference point and weighing factor, that are determined based on the subjective preferences and attitudes of decision makers toward risks. The reference point selection reflects the value function for gains and losses, unlike the Bayesian decision making based on the utility theory, and is one of the major advantages of the CPT. These merits can be useful in finding a more rational and reliable optimal alternative than the utility theory.

6. Conclusion

In this study, SMCDM was implemented to find an optimal ventilation strategy for the ERV using the stochastic predicted outputs of the BPS tool. In the building simulation domain, Bayesian decision-making based on the utility theory is generally used for handling SMCDM. This solves MCDM problems by reflecting the preferences of decision makers. However, the utility theory is not practical because it assumes that decision makers are rational beings. In contrast, the CPT proposed in this study can reflect (1) reference point setting, (2) diminishing sensitivity, and (3) loss aversion, and this is useful in solving the problem of the utility theory. In this study, the CPT was developed and the SMCDM of the ERV was conducted by selecting two ventilation strategies. In particular, reference points and

weighting factors were randomly selected for each scenario and their effects on deciding the optimal alternative were examined.

In the results, decision makers could obtain different total prospect values depending on the selection of reference point, which has considerable effect on the decision of the optimal alternative. It means that the decision-making results using the CPT can provide more realistic and trustworthy information compared to the utility theory.

Acknowledgements

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