

Hybrid Heat Pump Systems: Is Predictive Control Worth Using?

Patricia Ercoli – Università Politecnica delle Marche, Italy – p.ercoli@pm.univpm.it

Alice Mugnini – Università Politecnica delle Marche, Italy – a.mugnini@univpm.it

Fabio Polonara – Università Politecnica delle Marche, Italy – f.polonara@univpm.it

Alessia Arteconi – Università Politecnica delle Marche, Italy; & KU Leuven, Belgium – a.artecconi@univpm.it

Abstract

One of the possible solutions for renovating building heating systems is the use of hybrid systems, which consists of coupling heat pumps with traditional natural gas boilers. Hybrid Heat Pump systems are typically controlled to run the heat pump when the outside temperature is not too low, maintaining acceptable costs and good energy efficiency levels. However, when buildings also have a certain level of thermal inertia, proper management of the hybrid system can allow some flexibility. Especially in presence of non-programmable renewable sources, the control strategy can play an important role to maximize self-consumption.

The aim of this work is to assess the role of the control strategy in achieving this objective in relation to the cost reduction potential for energy bills. In particular, we investigate how much it is worth using an advanced control technique (e.g., a Model Predictive Control) compared to a Ruled Based Control to regulate the hybrid heating system of a residential building. The paper analyses a case study in which a building, equipped with Hybrid Heat Pump system assisted by photovoltaic panels serving a radiant floor, is controlled both through a Model Predictive and a designed Ruled Based Control. The objective of the controls is to minimize the energy bill for heating. The results are intended to assess whether the added complexity of the best performing model predictive control is justified by the magnitude of the performance increase that is obtained.

1. Introduction

In recent years, Heat Pumps (HPs) have seen an increase in their use in residential buildings. According to the International Energy Agency (IEA, 2021), as of 2015, there has been an upward trend in

HP sales within the European market, with an average annual growth rate of 12 %.

In the context of energy transition, HPs can offer a good solution for reducing energy consumption, as they give the possibility of using renewable energy sources such as aerothermal, geothermal and hydrothermal (Madonna et al., 2013), in addition to producing thermal energy through electricity (coming from the grid or produced on site).

The capability to correlate the thermal demand to electricity consumption is one of the most interesting aspects of HPs for unlocking the energy flexibility in buildings: the different levels of thermal inertia, which are already contained in buildings (thermal mass of the envelope or thermal storage devices), can be exploited to provide flexibility to the electricity grid.

One of the most frequently adopted solutions for exploiting the advantages of HPs, while maintaining acceptable costs and good levels of energy efficiency, are hybrid systems. In a Hybrid Heat Pump (HHP) system, the heat demand of the building is met by a HP coupled with a traditional boiler (EHI, 2020). This system is particularly useful in the presence of air-source HPs, since their performance depends heavily on the external climatic conditions. Although most HPs are installed in new constructions (IEA, 2021), hybrid systems present a good solution for home renovations (Dongellini et al., 2021). Indeed, in Italy, where the building stock is rather dated, the market of HHPs is one of the largest in the European Union, with about 7000 units sold in 2018 (EHI, 2020).

As for HP systems, a fundamental role is played by the control technique adopted also in HHP systems. To activate energy flexibility and optimize the ma-

nagement of sources, Model Predictive Controls (MPCs) are widespread. MPCs refer to an optimization problem to select the optimal set of control actions to minimize a given objective function at each time step.

There are many works available in the literature on the evaluation of the effectiveness of a MPC compared to a simpler Rule Based Control (RBC) for HPs. Fischer et al. (Fischer et al., 2017) have compared five different control methods, aimed at considering cases where the cost of electricity is constant, variable or cost-free in order to exploit self-consumption in a multi-family house equipped with an air-source HP supported by Photovoltaic (PV) panels and coupled to storage for domestic hot water. According to the authors, MPCs are more efficient than RBCs, with cost reductions of 6–16 % and 2–4 %, respectively. Zanetti et al. (2020) modeled an HHP, consisting of an air-to-water unit and a gas-fire boiler, assisted by PV panels and coupled to a water tank, serving a school supplied with floor heating. Comparing an RBC with an optimal control, from a thermal comfort point of view, the two controls provide similar results; regarding energy costs, the optimal control performs better as it allows savings of up to 20 % with an increase in self-consumption from 67 % (RBC) to almost 100 %. Ahmad et al. (Ahmad et al., 2013) have modeled a small house with an integrated HP via solar collector through a water tank for heating and hot water production. When compared to a simple RBC, the MPC was able to deliver savings of up to 9 %. From the comparison, the better performance of an optimized control is evident; however, as mentioned in (Fischer et al., 2017), its computational modeling and control fitting effort should be considered.

In this respect, this paper wants to analyze whether such advanced control techniques are worth using to control the heating system of a residential building according to a certain objective. In this regard, the paper proposes an analysis, in a simulation environment, of a case study in which a typical residential building subject to renovation, equipped with a HHP system assisted by PV panels and supplying a radiant floor, is controlled both through an MPC and a properly designed RBC. The main objective of the controls is to minimize the costs in

the electricity bill for heating.

2. Methodology

In order to assess the need for predictive control in an HHP system, the use of MPC and RBCs are compared in a residential building. In both cases, the controls aim to select the technology to be used (i.e., boiler or HP) to achieve economic savings, maximize the self-consumption of renewable sources and maintain thermal comfort. The comparison between MPC and RBC is carried out in a simulation environment. TRNSYS (TRNSYS 17, 2014) is selected to model the energy dynamics of the building. RBC is also modeled in TRNSYS, while MATLAB (MATLAB, 2014) is used for the MPC. The performance of RBC and MPC are evaluated by comparing the cost for satisfying the thermal demand of the building, the ability to maintain thermal comfort and the degree of exploitation of electricity produced by a PV plant installed on site. More details regarding the formulation of MPC and RBC are reported in the following subsections: subsection 2.1 describes the RBC, while in subsection 2.2 the formulation of MPC is explained.

2.1 Rule Based Control

RBC control is based on the determination of the external temperature (cut-off temperature) above which it is convenient to use the HP instead of the boiler. The cut-off temperature ($T_{\text{cut-off}}$) is determined through a comparison of the cost required to produce 1 kWh_{th}. For the HP, the cost is obtained considering the price of electricity withdrawal from the grid (c_E). To obtain the electrical energy absorbed by the HP, it was necessary to model the dependence of the COP on the temperatures of the air sources and the capacity ratio (CR). The model is based on the indications contained in EN 14825:2018 (CEN, 2018), starting from the performance map provided by the manufacturers. Since TRNSYS does not currently have a Type that allows modeling of a variable capacity HP, a new Type was developed by the authors, called Type 2701 (Ercoli et al., 2022). For the boiler, the cost of satisfying the heat demand is calculated by

multiplying the price of Natural Gas (c_{NG}) by its volume used. The latter is obtained by dividing the heat by the efficiency of the boiler (η_{BO}) and by the Higher Heating Value, HHV (condensing gas-fired boiler). Since also the availability of electricity from renewable sources is considered, two types of RBC were formulated: (i) base RBC (bRBC) and (ii) advanced RBC (aRBC). The bRBC is the simplest control where the switch between boiler and HP is determined only by $T_{cut-off}$. In addition, in the bRBC there is a thermostat that maintains the indoor air temperature (T_{air}) within a comfort range (2021 °C). On the other hand, the aRBC is set to force the HP to turn on regardless of $T_{cut-off}$ when a certain threshold of availability of PV is exceeded. It has been assumed as the minimum electrical power required for the minimum modulation of the HP (minimum CR of 0.3). To take advantage of the storage capacity of the heating system, the aRBC can exploit a wider comfort range (20 - 22 °C). However, in the absence of sufficient availability from PV, the HP and the boiler alternate their operation according to the set $T_{cut-off}$ and the thermostat is maintained within the 20 - 21 °C range.

2.2 Model Predictive Control

An MPC based on the system model was developed as an advanced control technique. The MPC can be divided into two parts: (i) the model of the system to be controlled and (ii) the optimizer. The system model (i) is responsible for forecasting the building's thermal demand. A lumped-parameter model based on the thermal-electricity analogy is used. Fig. 1 shows the structure of the resistances and capacitances (RC) network. It is composed of three thermal nodes. Each of them is represented by a capacitance (C) and a temperature (T). In particular, the thermal nodes represent the mass of the building envelope (C_e, T_e), the internal air (C_{air}, T_{air}) and the floor (C_f, T_f). The three thermal conductances K_{ea}, K_{fa} and K_{fg} model the heat flow between the three nodes, while the conductances K_w and K_{eo} model the heat flow between the external air (outdoor temperature, T_o) and T_{air} and T_e , respectively. The thermal flows entering the model are the solar gains (G_s) and the heating power provided by the heating system (Q_h). Since, as reported in Section 3,

the building is equipped with a radiant floor heating system, Q_H is directly applied to the thermal node of the floor (Fig. 1).

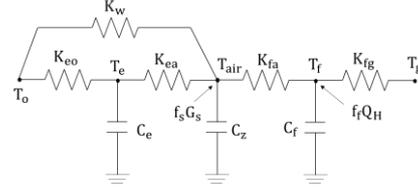


Fig. 1 – Third-order RC network building model

To obtain the numerical values of the parameters ($C_e, C_{air}, C_f, K_{ea}, K_{fa}, K_{fg}, K_w, K_{eo}, f_f$ and f_s), the model was trained starting from the data obtained from the building simulation in TRNSYS (Root Mean Square Error of 0.16 °C in the training period involving the whole month of January).

With this structure, the model can be represented with a discrete state space formulation (Eq. 1 and 2):

$$\mathbf{X}(k+\Delta k) = \mathbf{A} \cdot \mathbf{X}(k) + \mathbf{B} \cdot \mathbf{U}(k) \quad (1)$$

$$\mathbf{Y}(k+\Delta k) = \mathbf{C} \cdot \mathbf{X}(k) + \mathbf{D} \cdot \mathbf{U}(k) \quad (2)$$

with the vector $\mathbf{X} = [T_{air} T_e T_f]^T$, which represents the state of the system at each timestep k (Δk is the time interval between two timesteps), $\mathbf{U} = [T_o Q_H G_s]^T$ the input vector and \mathbf{Y} the vector contains the output (T_{air}). \mathbf{A} , \mathbf{B} , \mathbf{C} and \mathbf{D} are time-invariant real matrices depending on the parameters of the system.

The model, therefore, can simulate the thermal dynamics of the building. At this point, the optimizer (ii) must select the best control actions of the HHP system to maintain the T_{air} within an accepted comfort range. As in the case of aRBC (subsection 2.1), also in this case a greater tolerance is granted to the thermostat (20 - 22 °C) to increase the exploitation of the thermal inertia of the building. The control actions to be set are the control signals for the HP and the boiler (1/0 control signals: $ctrl_{HP}$ and $ctrl_{BO}$). The objective of the optimization is to minimize the energy bill over a forecast period (FP). To do this, a Linear Programming optimization problem was formulated (Eq. 3, 4, 5, 6 and 7).

$$\text{minimize} \sum_k^{FP} \left(\frac{Q_{HP}(k)}{COP(k)} \cdot c_E(k) + \frac{Q_{BO}(k)}{\eta_{BO} \cdot HHV} \cdot c_{NG} \right) \quad (3)$$

subject

$$\forall k = 1, \dots, FPT_{min} \leq T_{air}(k) \leq T_{max} \quad (4)$$

$$\forall k = 1, \dots, FP0 \leq Q_{HP}(k) \leq Q_{maxHP}(k) \quad (5)$$

$$\forall k = 1, \dots, FP0 \leq Q_{BO}(k) \leq Q_{maxBP} \quad (6)$$

$$\forall k = 1, \dots, FP Q_{BO}(k) + Q_{HP}(k) = Q_H(k) \quad (7)$$

Referring to Eq. 3, Q_{HP} and Q_{BO} are the thermal powers supplied by the HP and the boiler, respectively. These are the decision variables of the optimization problem. They can assume values between 0 and the maximum capacity of the HP (Q_{maxHP}) and the boiler (Q_{maxBO}), respectively. Eq. 4 contains the predictive model of the building. Eq. 5 and 6 set the boundary conditions for Q_{HP} and Q_{BO} . Finally, the constraint expressed by Eq. 7 is also inserted in order not to operate the HP and the boiler simultaneously (Q_H expresses the building load curve, Section 3). To incentivize the consumption of electricity produced by PV, a cost equal to 0 Eur kWh_e^{-1} is assigned to the electricity produced by PV. In general, Fig. 2 describes the dynamic behavior of the MPC. The MPC solves the optimization problem at each timestep k .

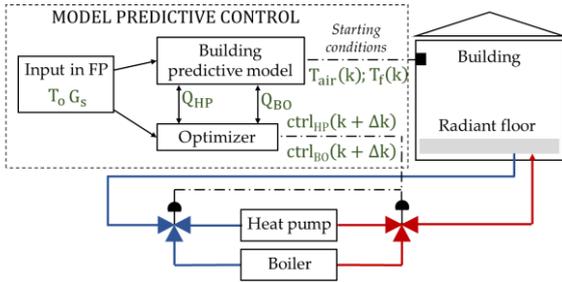


Fig. 2 – Schematic of the MPC

The actual temperatures (T_{air} and T_f) are passed as starting conditions to the MPC. Based on the receding horizon principle (Rawlings & Mayne, 2012), the MPC establishes the values of the control actions $ctrl_{HP}(k+\Delta k)$ and $ctrl_{BO}(k+\Delta k)$. These are derived from decision variables. Especially if $Q_{HP}(k+\Delta k)$ is greater than 0, $ctrl_{HP}(k+\Delta k)$ is 1, otherwise it is 0 (the same for $ctrl_{BO}$).

3. Case Study

A refurbishment for a residential building was considered as a case study. A single-family house whose construction characteristics refer to a period between 1991 and 2005 (Tabula Project (Corrado et al., 2014)) was chosen as an original building. The

building has a heated surface of 96 m^2 with a net heated volume of 299 m^3 . Table 1 contains the comparison between the thermal transmittances (U-values) of the original and the renovated building. Only the structures of the external walls and the windows were modified in the refurbishment (Table 1). In particular, the updated values were extrapolated from the most recent Italian regulation (DM, 2020).

Table 1 – U-values for renovated and original building

Building Status	Walls ($Wm^{-2}K^{-1}$)	Roof ($Wm^{-2}K^{-1}$)	Floor ($Wm^{-2}K^{-1}$)	Windows ($Wm^{-2}K^{-1}$)
Renovated	0.25	0.69	0.77	1.40
Original	0.59	0.69	0.77	1.70

For the simulations in TRNSYS, the climate file for a typical year of Ancona (43°37'N-13°31'E, Italy) was considered. Given an outdoor design temperature of -2 °C (UNI, 1976) and a T_{air} of 20 °C, the renovated building has a design peak load of 4.25 kW_{th} .

As mentioned, the heating system adopted in the renovated building is an HHP. It is composed of a modulating Air to Water Heat Pump (AWHP) and a condensing gas-fired boiler. To perform the study, commercial sizes of AWHP and boiler were taken as a baseline. For the HP, the operating characteristics were extrapolated from the data provided by a manufacturer. It presents a commercial size with 4.50 kW_{th} and 4.64 as COP, referred to an ambient temperature of 7 °C and a supply temperature of 35 °C. The boiler, on the other hand, has a capacity of 19 kW_{th} and an efficiency of 98 %, referred to the HHV (10.70 $kWh Sm^{-3}$).

As emission system, radiant floor heating was considered. The regulation of the heating system takes place with a compensation curve for the supply temperature (Fig. 3). The latter was calculated from the building load curve (Q_H) according to the T_o variation (Fig. 3).

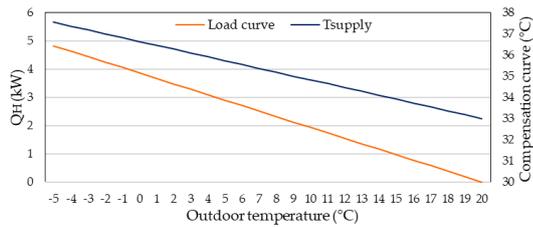


Fig. 3 – Building load curve (Q_H) and compensation curve

By applying the methodology described in Section 2.1 to the case study, the $T_{\text{cut-off}}$ obtained is 4°C . This was achieved by considering a natural gas cost of $1.225 \text{ Eur Sm}^{-3}$ and an electricity cost of $0.388 \text{ Eur kWh}^{-1}$ (ENEL, 2022). The renovated building was also equipped with a PV system installed on site. The PV plant consists of 12 monocrystalline silicon panels for a nominal peak power of 3.80 kW_e .

4. Results

A reference period was selected to compare the performance of MPC and RBC, i.e., the first two weeks of January. The analysis of the results will first be presented for the two RBC controls (4.1), to then be extended to the case of MPC (4.2). Finally, in section 4.3, we will try to answer the original question: “Is it worth using predictive control?”.

4.1 Results for RBCs

The comparison between the internal air temperature trend in the case of bRBC and aRBC is shown in Figs. 4 and 5. In both bRBC and aRBC, there is a certain period in which the air temperature does not respect the thresholds set on the thermostat (Section 2.1). In the case of bRBC (Fig. 4), the air temperature drops below the minimum threshold (i.e., 20°C) for the 2 % of the time (7 hours), reaching a minimum of 19.43°C . From this point of view, better behavior is obtained with the aRBC. In fact, looking at Fig. 5, the air temperature assumes values lower than 20°C for 1 hr and 30 mins (0.45 % of the time), reaching a minimum of 19.93°C .

As for the upper temperature threshold, this is different between the two controls when PV is available. In fact, the aRBC can exploit the flexibility of the thermostat to increase PV self-consumption

and reach 22°C . Comparing Fig. 4 and 5, it can be noted that there is a greater exploitation of the upper band granted to the thermostat in case of aRBC. Indeed, the average air temperature in case of bRBC is 20.47°C , while it becomes 20.62°C with the aRBC. Furthermore, with the aRBC, the air temperature exceeded the upper limit of 22°C for a time of 2 hrs and 45 mins (about 0.82 % of the time) with a peak of 22.48°C .

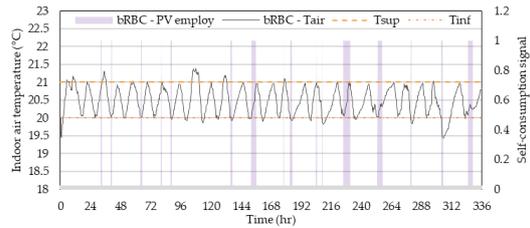


Fig. 4 – Internal air temperature (T_{air}) and self-consumption signals during basic RBC (bRBC) use

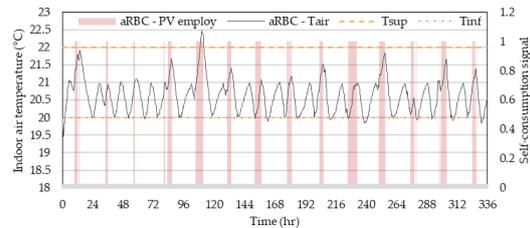


Fig. 5 – Internal air temperature (T_{air}) and self-consumption signals during advanced RBC (aRBC) use

Figs. 6 and 7 show the involvement of the single technologies (AWHP and boiler) in the case of the RBCs. From the comparison of Figs. 6 and 7, with bRBC, there is a higher utilization of the boiler and higher thermal demand peaks than in the case where the aRBC is used. In addition, from Fig. 7, there is an increase in AWHP utilization through use of the aRBC, since, as described in Section 2.1, the control is set to force the AWHP to turn on regardless of $T_{\text{cut-off}}$. In particular, the AWHP is found to be operating for 141 hrs and 45 mins through aRBC, 6.98 % more than bRBC. The boiler, on the other hand, works 7.17 % less during use of the aRBC than the basic one, for a total time of 61 hrs and 30 mins. Also, comparing Figs. 6 and 7, in addition to an increase of AWHP application, it is also possible to see a slight increase in the PV self-consumption by aRBC.

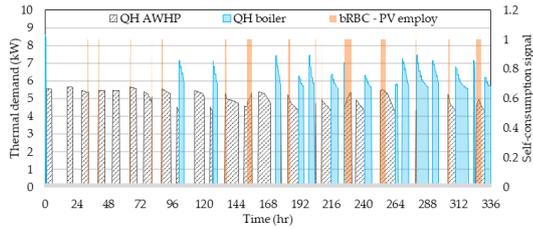


Fig. 6 – Thermal demand (Q_H) trends and self-consumption signals during bRBC use

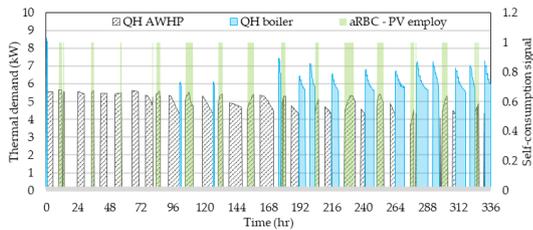


Fig. 7 – Thermal demand (Q_H) trends and self-consumption signals during aRBC use

The difference in utilization between the two RBCs can also be seen through the thermal demand via Fig. 8; from the comparison with the bRBC, the use of the aRBC involves an increase of the thermal demand of 6.46 % covered by the AWHP and a decrease of 7.18 % for the contribution of the boiler.

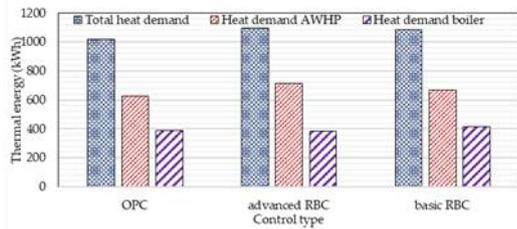


Fig. 8 – Thermal demand of the hybrid system, AWHP and boiler during the application of the three controls

The differences in terms of PV exploitation are also highlighted in Fig. 9, where the self-consumption of electric energy during advanced control is 6.43% higher than that through bRBC.

In terms of performance, the use of aRBC results in a slight reduction of 0.26 % in average COP, with a value of 4.21 compared with that of the bRBC of 4.22. Despite the slight decrease in average COP, the lower boiler utilization by the aRBC resulted in lower energy bill costs. From Fig. 10, a net saving is achieved through aRBC. In particular, the costs due to the boiler, which, compared with the case of the bRBC, decreased by 76.8 %.

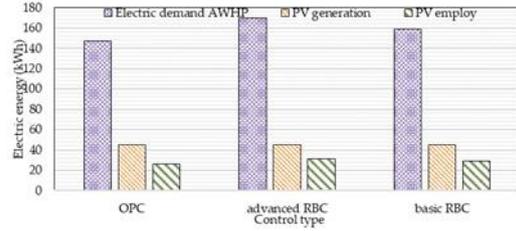


Fig. 9 – Electric consumption required by the AWHP, generated by the PV panels and self-consumed during the application of the three controls

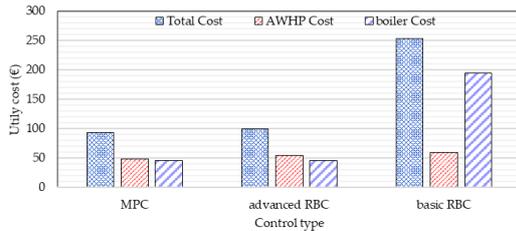


Fig. 10 – Cost due to use of hybrid system, AWHP and boiler during the application of the three types of control

4.2 Results for MPC

As with the two RBCs, Fig. 11 shows the trend of the internal air temperature in the reference period for the MPC. It can be noted that the temperature fluctuates frequently around 20 °C with an average value of 20.11 °C. The MPC is able to maintain the air temperature within a narrower range of variation. Both violations towards the lower limit of the thermostat and towards the upper one are reduced in comparison with the RBCs. In fact, as can be seen in Fig. 11, the upper limit of 22 °C is not exceeded, and the air temperature reaches a peak of 21.4 °C.

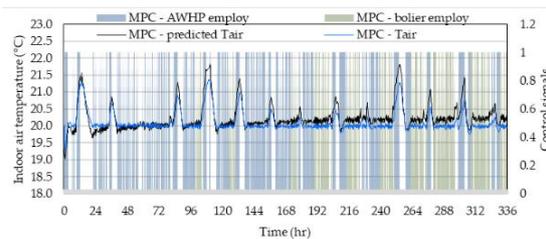


Fig. 11 – Calculated and predicted indoor temperature trends and AWHP and boiler usage signals during MPC application

In terms of running time, the use of the MPC leads to an overall reduction; both the AWHP and the boiler are used for less time, meaning 123 hr and 45 min and 58 hrs and 15 mins, respectively. Overall, there is a reduction in system utilization of less than 10.46 % compared with the aRBC and less than 8.43 % compared with the bRBC. Through Fig. 8, it

can be seen that the use of the MPC also leads to 7.19 % in thermal demand reduction due to the entire hybrid system compared with the aRBC. Even the AWHP provides a lower thermal demand of 11.82 %, probably due to its decreased use. In addition, this could lead to a reduction of the self-consumption, as shown in Fig. 10; in fact, compared with the aRBC there is a reduction of 17.33 % in self-consumption. Moreover, the AWHP, through the MPC, is able to provide better performance, with 4.26 as average COP, 0.94 % higher than the bRBC. Considering the energy costs, the MPC performs better than the RBCs, given the bill savings objective (Fig. 11). In fact, within a two-week reference period, the use of MPC, in the case study considered, resulted in savings of 63.27 % compared with bRBC and 5.93 % compared with aRBC.

4.3 Is It Worth Using Predictive Control?

The results shown in the previous sections showed that MPC achieved better performance in terms of cost savings and concerning the thermostat. On the other hand, the best performance regarding self-consumption of PV was obtained from the advanced RBC.

What is important to note is that, although the MPC is better at achieving the objective (e.g., cost reduction), modeling difficulties not present in RBC cases should be taken into account. In fact, the MPC being a model-based control, in the case of incorrect or missing data, there could be a wrong estimation of the thermal demand of the building and, consequently, incorrect decision making. Furthermore, the modeling and implementation difficulty that MPC requires compared with RBCs cannot be overlooked.

In the case studied, the advanced RBC turned out to be a good compromise; in fact, it was possible to achieve good savings over the basic RBC with less effort than the MPC. In particular, this was possible through: (i) a good estimation of the $T_{\text{cut-off}}$ (e.g. using the method described in section 2.1), (ii) forcing the AWHP to turn on during PV generation and (iii) the activation of the flexibility of thermostat. Indeed, it is possible to see in Fig. 12 how the $T_{\text{cut-off}}$ varies while using the MPC in com-

parison with the fixed 4 °C of RBCs. From the comparison, it can be seen that this transition temperature between the two generators was properly estimated through a comparison of the cost required to produce 1 kWh_{th} (section 2.1).

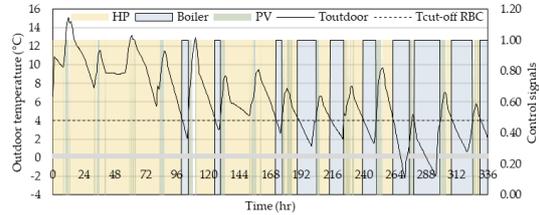


Fig. 12 – Variation of the outdoor temperature and AWHP and boiler use convenience signals based on cost

5. Conclusion

In this paper, we asked whether it is worth using a MPC to control a residential hybrid heating system with PV panels. To do this, a MPC was compared with two RBCs, one basic and one advanced.

From the results, the main conclusions can be summarized in the following points:

- The MPC is more effective in reducing the energy cost: a saving of 63.27 % was estimated in relation to the basic RBC and 5.93 % compared to advanced RBC.
- The advanced RBC allows a higher self-consumption of PV compared with MPC to be obtained (with the MPC a reduction in self-consumption of 17.33 % was achieved in comparison with advanced RBC).
- With the MPC there is no violation of the upper band of the thermostat – a phenomenon that occurs with the advanced RBC (0.82 % of the time, 2 hrs and 45 mins).

Although MPC has shown better performance than RBCs in terms of comfort and savings, its application for a system such as the one analysed should also consider the level of difficulty that its implementation requires. Through advanced RBC, it was possible to achieve good savings over the basic RBC but with less effort than the MPC, thus offering a good compromise between the two controls.

Despite good results from both MPC and advanced RBC, there were still unused amounts of self-

generated electrical power. One way to mitigate wasted electrical power could be to introduce additional integration devices to increase system flexibility and further exploit the building's storage capabilities. In this context, it might be interesting to further explore the comparison between different types of control.

References

- Ahmad, M. W., M. Eftekhari, T. Steffen and A.M. Danjuma. 2013. "Investigating the performance of a combined solar system with heat pump for houses" *Energy and Buildings* 63: 138-146. doi: <https://doi.org/10.1016/j.enbuild.2013.03.055>
- CEN (European Committee for Standardization). 2018. *EN 14825:2018. Air Conditioners, Liquid Chilling Packages and Heat Pumps, with Electrically Driven Compressors, for Space Heating and Cooling – Testing and Rating at Part Load Conditions and Calculation of Seasonal Performance*
- Corrado, V., I. Ballarini, and S. P. Corgnati. 2014. "Building Typology Brochure – Italy" EPISCOPE. Accessed January 10. <https://episcopes.eu/communication/download/>
- Dongellini, M., C. Naldi, and G. L. Morini. 2021. "Influence of sizing strategy and control rules on the energy saving potential of heat pump hybrid systems in a residential building" *Energy Conversion and Management* 235. doi: <https://doi.org/10.1016/j.enconman.2021.114022>.
- EHI (European Heating Industry). 2020. "Heating Market Report 2020" EHI. Accessed March 30. <https://ehi.eu/heating-market-report/heating-market-report-2020/>
- ENEL, 2022. www.enel.it/?ecid=paidsearch-google-always_on_top_keyword-brand-enel_exact&gclid=EAlaIqobChMI5sa8hvaf9wIVBLLVCh0o4wDQEAAAYASAAEgJAH_D_BwE&gclsrc=aw.ds
- Ercoli, P., A. Mugnini and A. Arteconi. 2022. github.com/diismunivpm/Type2701-for-TRNSYS.git
- Fischer, D., J. Bernhardt, H. Madani, and C. Wittwer. 2017. "Comparison of control approaches for variable speed air source heat pumps considering time variable electricity prices and PV." *Applied Energy* 204: 93-105. doi: <https://doi.org/10.1016/j.apenergy.2017.06.110>
- IEA - International Energy Agency. 2021. "Heat Pumps" IEA. Accessed March 21. <https://www.iea.org/reports/heat-pumps>
- Madonna, F., and F. Bazzocchi. 2013. "Annual performances of reversible air-to-water heat pumps in small residential buildings" *Energy and Buildings* 65: 299-309. doi: <https://doi.org/10.1016/j.enbuild.2013.06.016>.
- MATLAB - Matrix Laboratory. 2014. Version R2014a, MathWorks, Natick, Massachusetts, USA.
- Ministero dello sviluppo economico. 2020. "DECRETO 6 agosto 2020 - cd ECOBONUS (In Italian)" Ministero dello sviluppo economico. Accessed January 10. <https://www.gazzettaufficiale.it/eli/id/2020/10/05/20A05394/sg>
- Rawlings, J. B., and D. Q. Mayne. 2012. *Model Predictive Control: Theory and Design*. ISBN 9780975937709.
- TRNSYS 17 - Transient System Simulation Tool. 2014. Version 17.2, Solar Energy Laboratory, University of Wisconsin, Madison, USA.
- UNI (Italian National Agency for Unification). 1976. *UNI 5364:1976. Hot water heating systems. Rules for presentation of offer and for testing*.
- Zanetti, E., M. Aprile, D. Kum, R. Scoccia, and M. Motta. 2020. "Energy saving potentials of a photovoltaic assisted heat pump for hybrid building heating system via optimal control." *Journal of Building Engineering* 27. doi: <https://doi.org/10.1016/j.job.2019.100854>