Numerical Investigation of Radiation Efficiency of a Cross-Laminated Timber Floor

Marco Caniato – Free University of Bozen-Bolzano, Italy – marco.caniato@unibz.it
Nicola Granzotto – Free University of Bozen-Bolzano, Italy – nicolagranzotto74@gmail.com
Federica Bettarello – University of Trieste, Italy – fbettarello@units.it
Arianna Marzi – Free University of Bozen-Bolzano, Italy – arianna.marzi@natec.unibz.it
Paolo Bonfiglio – Materiacustica srl, Italy – paolo.bonfiglio@materiacustica.it
Andrea Gasparella – Free University of Bozen-Bolzano, Italy – andrea.gasparella@unibz.it

Abstract

Cross-Laminated Timber (CLT) is a building technology that is becoming increasingly popular due to its sustainability and availability. Nevertheless, CLT structures present some challenges, especially in terms of both structure-borne and airborne insulation. In this paper, a 200-mm CLT floor was characterized in the laboratory, according to ISO standards, by using a standard tapping machine in order to understand its vibro-acoustic behavior in terms of radiation efficiency for structural excitation. In particular, experimental tests were compared to analytical prediction models available in the literature to check the accuracy of simulation methods in the prediction of the radiation capability of CLT structures.

1. Introduction

In recent years, the use of timber as a construction material in the building sector has been increasing. Sustainable edifices made exclusively with timber or refurbishment of conventional heavyweight houses using new timber structures are common in most cities. In view of this, the use of wooden components and, in particular, Cross Laminated Timber (CLT) elements has greatly increased in the past decade. Timber has a number of advantages: it is an eco-friendly material, well suited for thermal comfort and a fast-track on-site construction process, featuring the possibility of implementing existing structures thanks to its reduced weight.

However, its acoustic simulations lack a complete description, since the literature does not always provide reliable methods capable of predicting reliable values as regards acoustic performance. From this perspective, further studies are needed to develop and improve prediction models of CLT floor sound and vibrational behavior (Yang et al., 2021; Zhang et al., 2020). Among all available parameters, one worthy of investigation is represented by radiation efficiency. Indeed, the sound radiation index is of paramount importance for understanding and simulating the behavior of these elements (Kohrmann, 2017).

A recent study (Jansson, 2021) has shown how the use of software and calculation models, currently available for the study of the characteristics of multilayer systems, are not reliable when wooden structures are used. For this reason, it is of paramount importance not only to know the characteristics of the materials, but also their radiation efficiency. However, the study of this parameter, in relation to the characterization of CLT floors, is still partially incomplete.

Hence, to design CLT structures with good acoustic insulation, there is the need to characterize the sound radiation of the vibrating elements. The radiation efficiency can be computed using dedicated equations or simulated using Finite Elements Methods. In this paper, this latter approach is used to verify if, with reference CLT floor, the method can be used and if it could provide reliable and robust results.
2. Material and Methods

2.1 Description of Investigated CLT Floor

This paper presents the results of a numerical investigation of the radiation efficiency of a CLT floor with a thickness of 200 mm and a size of 4155 mm x 3000 mm, measured in a laboratory built in accordance with the ISO 10140 series standards, using a standardized tapping machine (B&K 3297) as mechanical source (Fig. 1).

![Investigated CLT floor mechanically excited using a standardized tapping machine](image1)

The laboratory was built to minimize flanking transmission and, in particular, it features a volume of transmitting room of 50.9 m³ and a volume of the receiving room of 60.63 m³.

![Picture of the available acoustic facility](image2)

A grid of accelerometers was used to monitor the acceleration levels. Precisely, the measurement pattern is described in Fig. 3.

![Tapping machine positions](image3)

The radiation efficiency was measured using following expression:

\[
\sigma_{rad} = \frac{W_{rad}}{\rho_0 c_0 S \langle v^2 \rangle_{SL}}
\]

(1)

where \(W_{rad} [W]\) is the radiation power, \(\rho_0 [kg/m^3]\) is the air density, \(c_0 [m/s]\) is the speed of sound, \(S [m^2]\) is the floor surface, \(\langle v^2 \rangle_{SL}\) is the average square vibration velocity on the receiving side. The radiation power was evaluated using the average sound pressure level (Svantek 958) and the reverberation time in the receiving room. The averaged squared velocity was measured using accelerometers (Dytran 3023) mounted on the bottom side of the CLT floor.

2.2 Description of Numerical Approaches

As previously described, CLT floor is made of five layers (having a thickness of 4 cm each and density of around 420 kg/m³). It is well known in the literature that each layer is an orthotropic solid and a comprehensive analysis requires the knowledge of 9 independent parameters (3 Young’s modulus, 3 shear modulus and three Poisson’s ratio). Table 1 summarizes values of mechanical parameters utilized as input data of investigated numerical approaches.
Parameters in Table 1 were used for layers 1, 3 and 5. Layers 2 and 4 were modeled using the same parameters in a 90-degree-rotated coordinate system. In the following sessions, two different numerical approaches will be described.

<table>
<thead>
<tr>
<th>$E$ [GPa]</th>
<th>$G$ [GPa]</th>
<th>$\nu$ [-]</th>
<th>$\eta$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1=11</td>
<td>G12=0.69</td>
<td>0.14</td>
<td>0.01</td>
</tr>
<tr>
<td>E2=0.37</td>
<td>G13=0.069</td>
<td>0.33</td>
<td>0.01</td>
</tr>
<tr>
<td>E3=0.37</td>
<td>G23=0.69</td>
<td>0.14</td>
<td>0.01</td>
</tr>
</tbody>
</table>

2.2.1 Hybrid FEM-Analytical model I

A statistical radiation efficiency model was implemented, based on the modal-average formulations, using frequency-dependent stiffness properties as input data. Such a statistical approach requires some additional assumptions: (a) high modal density and modal overlap over the entire frequency range (b) the sound power is only radiated by resonant modes; (c) the resonant modes are uncorrelated; (d) equipartition of modal energy can be applied. The radiation efficiency can be calculated as:

$$\sigma_{\text{ortho}}(\omega) = \frac{L_xL_y}{\pi^2\eta_d} \int_0^{\pi/2} \sigma(\omega, \phi)k_{B}\frac{\partial k_{B}}{\partial \phi} d\phi. \quad (2)$$

where $L_x$ and $L_y$ are the lateral size of the CLT floor, $\sigma(\omega, \phi)$ the radiation efficiency calculated using Leppington’s formulation (Leppington et al., 1982), $\eta_d$ the plate modal density and $k_B$ the structural bending wave propagating in the plate. At any propagation angle $\phi$, the direction-dependent bending wavenumber can be estimated from the wavenumber components along the principal directions $k_{Bx}$ and $k_{By}$ by applying a well-established orthotropic elliptic model.

To summarize, if the wavenumber components, along with the principal directions $k_{Bx}$ and $k_{By}$ are known together with the size and the plate density, the radiation efficiency can be calculated.

In the present research, the wavenumbers $k_{Bx}$ and $k_{By}$ were determined using a simplified finite element model (implemented in Comsol Multiphysics) and the Inhomogeneous Wave Correlation (IWC). In particular, two finite element models of freely suspended CLT beams (1 m long) in x and y directions are solved when a unit force in z direction is exerted on a side of the beam.

Once each model is solved, the $z$ direction complex displacement is computed along each beam on a set of equally spaced points (1 cm of spacing has been considered). By applying the IWC method, it is possible to determine the dispersion relation (the wavenumber as a function of the frequency) in both principal directions (Fig. 5).
2.2.2 Hybrid FEM-Analytical model II
The second hybrid method requires a two-step procedure:
1. A FEM model of the entire floor is solved for a mechanical point excitation. In this case, the three different simulations were considered according to the positions of the tapping machine during experimental.
2. Once the model is solved, the z direction complex velocity is computed on a grid of equally spaced points (5 cm of spacing was considered). The mean-squared velocity in Eq. 1 can be directly calculated as the average of the squared velocity, while radiated power of Eq. 1 has been computed using the Discrete Calculation Method (DCM) (Santoni et al., 2019):

\[ W_{rad} = \sum_i \left[ \text{Re}(Z_{ii}) \| v_i \|^2 + \sum_j \text{Re} \left( Z_{ij} v_i v_j^* \right) \right]. \quad (3) \]

where \( Z_{ii} \) and \( Z_{ij} \) are the self- and cross-radiation impedances, respectively.

2.2.3 Accuracy
In order to estimate the quality of the fit, a standard deviation is calculated by taking into consideration the measured values as the average data (\( \mu \)) for each of \( n \) frequency bands (1/3 octave) and the calculated values as experimental ones (Eq. 3):

\[ \sigma_{dev} = \sqrt{\frac{1}{N} \sum_{i=1}^{n} (x_i - \mu)^2} \quad (3) \]

In addition, the mean difference is computed (Eq. 4):

\[ \Delta_{mean} = \frac{1}{N} \sum_{i=1}^{n} |x_i - \mu| \quad (4) \]

3. Results
Experimental radiated sound power and structural velocity levels are shown in Fig. 6.

From the curves in the previous graph, it is possible to observe satisfactory accuracy of both simulation techniques. The average absolute differences are equal to 1 dB and 1.8 dB for Model I and Model II when compared with experimental tests. In particular, model II is able to investigate modal behavior of the plate. Fig. 8 depicts the modal resonances of the full floor at frequencies of up to around 150 Hz.
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Furthermore, deviations (i.e., arithmetic differences) between experimental data and numerical simulations are depicted in Fig. 9. From the comparison, it is possible to observe a better capability of Model I to simulate the radiation efficiency of the CLT structure, while both methodologies show higher deviations at frequencies lower than 125 Hz, which is the frequency region governed by resonant modes of the floor and it is highly dependent on boundary conditions, generally difficult to implement in simplified numerical models.

In Table 2, the simulation accuracy is reported. It can be noticed that model I presents better values than model II, mostly because of low frequency values differences reported in Fig. 9.

Table 2 – Accuracy for different numerical methods

<table>
<thead>
<tr>
<th>Numerical method</th>
<th>Dev.st</th>
<th>Δmean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model I</td>
<td>0.91</td>
<td>0.74</td>
</tr>
<tr>
<td>Model II</td>
<td>1.72</td>
<td>1.47</td>
</tr>
</tbody>
</table>

4. Conclusion

In this paper, two different numerical approaches were utilized for the prediction of the radiation efficiency of a Cross-Laminated Timber frame for building constructions. Results were compared with experimental tests carried out in a dedicated laboratory. The accuracy of proposed methodologies was proved to be between 1 and 2 dB in terms of average radiation efficiency level. The implementation of both methodologies is straightforward and requires knowledge of the mechanical properties of the orthotropic timber material. Future work will be devoted to extension of the proposed numerical formulations to different acoustic and mechanical excitations.
Acknowledgement

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Nomenclature

Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$W_{rad}$</td>
<td>radiation power (W)</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>air density (kg/m$^3$)</td>
</tr>
<tr>
<td>$c_0$</td>
<td>speed of sound (m/s)</td>
</tr>
<tr>
<td>$S$</td>
<td>floor surface (m$^2$)</td>
</tr>
<tr>
<td>$&lt;\ddot{v}_x&gt;_t$</td>
<td>is the average square vibration velocity on the receiving side (m/s)</td>
</tr>
<tr>
<td>$E$</td>
<td>Young’s modulus (GPa)</td>
</tr>
<tr>
<td>$G$</td>
<td>Shear modulus (GPa)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poissino ratio (-)</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Loss factor (-)</td>
</tr>
<tr>
<td>$L_x$</td>
<td>dimension (m)</td>
</tr>
<tr>
<td>$L_y$</td>
<td>y-direction size dimension (m)</td>
</tr>
<tr>
<td>$\sigma(\omega,\phi)$</td>
<td>radiation efficiency (-)</td>
</tr>
<tr>
<td>$\eta_s$</td>
<td>plate modal density (-)</td>
</tr>
<tr>
<td>$k_B$</td>
<td>structural bending wave propagating in the plate (m$^{-1}$)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>propagation angle (rad)</td>
</tr>
<tr>
<td>$Z$</td>
<td>radiation impedances (rayls)</td>
</tr>
</tbody>
</table>

Subscripts/Superscripts

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>x-direction</td>
</tr>
<tr>
<td>y</td>
<td>y-direction</td>
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</table>

References


