Technical Appendix to:

The Burden of Unanticipated Inflation:

Analysis of an Overlapping Generations Model with Progressive Income Taxation and Staggered Prices

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1 Introduction

This Appendix provides several versions of the overlapping generations model (OLG) and representative agent (RA) model considered in the main text. Besides the details of the respective model we also present further results to check the robustness of our findings.

If not mentioned otherwise the definition of the variables follows the one in the main text.

The next section gives a detailed account of the demographic structure which underlies our OLG model. Section 3 develops a simple OLG version of the familiar representative agent Ramsey model. The solution of this model provides starting values for the solution of a more elaborate OLG model with endogenous labor supply and progressive income taxation in Section 4. Section 5 introduces money into this model and Section 6 adds nominal frictions.

2 Demographic Structure

In our model periods correspond to quarters. In each period there are T generations alive. They work the first $R-1$ periods of their life and retire at age R. The size of each generation s is ψ_s. A member of generation s survives with probability ϕ_s to age $s+1$ so that the mass of generation $s+1$ is given by

$$\psi_{s+1} = \phi_s \psi_s. \tag{1}$$

We normalize the total mass $\sum_{s=1}^{T} \psi_s$ to unity.

We calibrate the survival rates from the age-specific death rates for the total population in the US in the year 2000. Linear interpolation between the annual survival rates for the 20 through 79 year old population provides the quarterly rates. We assume that all members of the first generation $s = 1$ survive and that all members of the last generation $s = 240$ die.

Figure 1 displays the survival rates (before normalization). The dots correspond to the annual survival rates from Arias (2002).

Each generation s consists of m different productivity groups of mass ν_h, $\sum_{h=1}^{m} \nu_h = 1$. Each new born agent is assigned to one of these groups and remains in this group during his lifetime. His productivity $\epsilon_{s,h} = e_s z_h$ has an age and an individual specific

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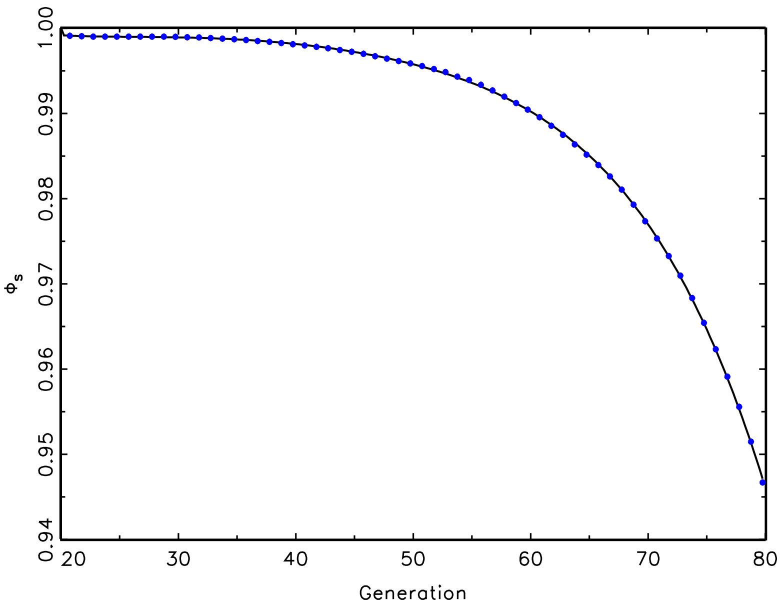
Figure 1: Survival Rates



component, e_s and z_h, respectively. We use the age specific productivity profile from Hansen (1993) and interpolate linearly to obtain a quarterly series. Figure 2 displays this profile. The dots correspond to Hansen's (1993) data for the 21 through 60 year old workers. The quarterly rates are normalized so that average productivity equals unity.[1]

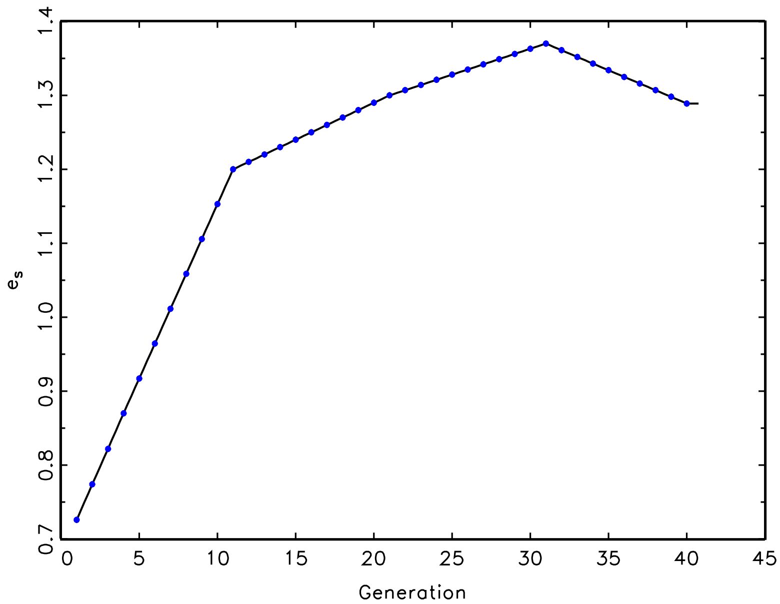
We assume that the group specific productivity z_h is log-normally distributed around the age $s = 1$ mean productivity with standard deviation σ_z. We approximate this distribution at m equally spaced points, choose σ_z so as to match the empirically observed Gini ratio of income, and normalize the weights so that $\sum_{h=1}^{m} \nu_h z_h = 1$.

[1] Let $\tilde{e_s}$ denote the original efficiency factor of generation s and $mw = \sum_{s=1}^{R-1} \psi_t$ the mass of workers. The normalized efficiency factors e_s are defined as

$$e_s = \frac{\tilde{e}_s}{\sum_{s=1}^{R-1} \frac{\psi_s}{mw} \tilde{e}_s}$$

so that $\sum_{s=1}^{R-1} \frac{\psi_s}{mw} e_s = 1$.

Figure 2: Age Specific Producitivity Profile



3 OLG Version of the Stochastic Growth Model

Our first model most closely resembles the stochastic growth model. There are three sectors: households, firms, and the government. We describe them in turn.

3.1 Households

The household sector consists of $T \times m$ different types of agents with mass $\psi_s \nu_h$ as described in Section 2. Agents that die at the end of period t leave bequests. We assume that the government confiscates all bequests. Section 3.5 deals with perfect annuity markets.

At at calendar time t an agent of age $s \in \{1, 2, \ldots, T\}$, who belongs to productivity group $h \in 1, 2, \ldots, m$, maximizes his expected life-time utility

$$U_{t,s,h} := \mathbb{E}_t \left[\sum_{j=s}^{T} \beta^{j-s} \prod_{i=s}^{j} \phi_i \left(\frac{c_{t_j,s,h}^{\gamma(1-\sigma)} - 1}{1-\sigma} \right) \right] \tag{2}$$

subject to the sequence of budget constraints

$$
\begin{aligned}
j &= s, s+1, \dots, T: \\
c_{t_j,s,h} &= (1-\tau)\left[e_j z_h w_{t_j} n + (r_{t_j} - \delta) k_{t_j,s,h} + \omega_{t_j,s,h}\right] + pens_{t_j,s,h} + trs_{t_j} \\
&\quad + k_{t_j,s,h} - k_{t_j+1,s+1,h}, \\
e_j &= 0 \text{ for } j = R, R+1, \dots, T, \\
pens_{t_j,s,h} &= 0 \text{ for } j = 1, 2, \dots, R-1, \\
k_{t_j,s,h} &= 0 \text{ for } j = 1, \\
k_{t_j+1,s+1,h} &= 0 \text{ for } j = T, \\
t_j &:= t + j - s.
\end{aligned}
\tag{3}
$$

BCO

$c_{t_j,s,h}$ denotes his consumption, n his exogenously given supply of working hours, w_{t_j} and r_{t_j} denote the wage rate and the interest rate, respectively, δ is rate at which physical capital depreciates, $k_{t_j,s,h}$ are the agent's assets, τ is the tax rate, $\omega_{t_j,s,h}$ is profit income, trs_{t_j} are lump sum government transfers, and $pens_{t_j,s,h}$ denotes pensions which are paid to retired agents. Note that variables without index t_j, s, or h are assumed to be constant across time, age, or productivity group, respectively.

We assume that aggregate profits Ω_t are distributed to households according to their shares in aggregate wealth. Yet, because younger households may find it optimal to lend against future income, we restrict the distribution to households with positive assets. Furthermore, since our solution procedure (log-linearization at the balanced growth path) cannot deal with abrupt changes from negative to positive wealth, we use the shares that apply in the stationary equilibrium of our model. Thus, the shares of aggregate profits Ω_t received by a household of age s and productivity type h equal

$$
\begin{aligned}
\kappa_{s,h} &= \frac{\max\{0, k_{s,h}\}}{\bar{K}} \\
\bar{K} &= \sum_{s=1}^{T} \sum_{h=1}^{m} \psi_s \nu_h \max\{0, k_{s,h}\}.
\end{aligned}
\tag{4}
$$

A_Share

This guarantees that the shares ad up to unity.

Pensions are proportional to the average net wage income received by working agents in group h in the stationary equilibrium:

$$
pens_{t_j,h} := pens_h = \zeta(1-\tau) \underbrace{\sum_{s=1}^{R-1} \frac{\psi_s}{\sum_{s=1}^{R} \psi_s} e_s}_{=1} z_h w n = \zeta(1-\tau) z_h w n, \tag{5}
$$

pens

where ζ is the replacement rate.

At time t the first-order conditions of all living agents of type h consist of their respective budget constraint (3) [BCO] at $j=s$ and the following $2T-1$ equations: [FOC1]

$$
\begin{aligned}
s &= 1,\dots,T: & \lambda_{t,s,h} &= \gamma c_{t,s,h}^{\gamma(1-\sigma)-1}, && \text{(6a) [FOC1a]}\\
s &= 1,\dots,T-1: & \lambda_{t,s,h} &= \mathbb{E}_t \lambda_{t+1,s+1,h}\beta\phi_{s+1}(1+(1-\tau)(r_{t+1}-\delta)), && \text{(6b) [FOC1b]}
\end{aligned}
$$

where $\lambda_{t,s,h}$ is the Lagrange multiplier attached to the budget constraint at time t for the agent of age s and of productivity type h.

On a balanced growth path, wages, interest rates, profits, bequests, transfers and pensions are constant, and individual variables only depend on age s and type h but not on calendar time t. In this case the system of $3T-1$ equations (3) [BCO] and (6) [FOC1] can be reduced to the following system of linear equations in the $T-1$ unknowns $k_{s,h}$, $s=2,3,\dots,T$:

$$
\begin{bmatrix} b_{1,h} \\ b_{2,h} \\ b_{3,h} \\ \vdots \\ b_{T-1,h} \end{bmatrix}
=
\begin{bmatrix}
a_{11} & a_{12} & 0 & 0 & 0 & \dots & 0 & 0 \\
a_{21} & a_{22} & a_{23} & 0 & 0 & \dots & 0 & 0 \\
0 & a_{32} & a_{33} & a_{34} & 0 & \dots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & a_{T-1,T-2} & a_{T-1,T-1}
\end{bmatrix}
\begin{bmatrix} k_{2,h} \\ k_{3,h} \\ k_{4,h} \\ \vdots \\ k_{T,h} \end{bmatrix}, \tag{7}
$$
[Sys1]

$$
\begin{aligned}
a_{s+1,s} &= -A, \quad A := 1+(1-\tau)(r-\delta),\\
a_{s,s} &= 1+AB_s, \quad B_s := [\beta\phi_{s+1}(1+(1-\tau)(r-\delta))]^{\frac{1}{\gamma(1-\sigma)-1}},\\
a_{s,s+1} &= -B_s,\\
b_{s,h} &= (1-\tau)wn\,(e_s z_h - B_s e_{s+1} z_h) + (1-B_s)trs\\
&\quad + (1-\tau)(\omega_{sh} - B_s\omega_{s+1h}),\\
&\quad s = 1,\dots,R-1\\
b_{R-1,h} &= (1-\tau)e_{R-1h} z_h wn + +(1-B_{R-1})trs\\
&\quad + (1-\tau)(\omega_{R-1h} - B_{R-1}\omega_{Rh}) - B_{R-1}pens_h,\\
b_{s,h} &= (1-B_s)(trs + pens_h) + (1-\tau)(\omega_{sh} - B_s\omega_{s+1h}),\\
&\quad s = R,\dots,T-1.
\end{aligned}
$$

3.2 Production [Psector]

There are two sectors of production. The final good Y_t is assembled from a unit mass of differentiated goods $Y_t(j)$, $j\in[0,1]$ according to

$$
Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon}{\epsilon-1}} dj\right)^{\frac{\epsilon-1}{\epsilon}}, \quad \epsilon > 1. \tag{8}
$$
[Final]

Profit maximization implies the demand functions:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t \tag{9}$$ demand

and the price index

$$P_t = \left(\int_0^1 P_t(j)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}, \tag{10}$$ pindex

where $P_t(j)$ denotes the nominal price of good j.

Each of the differentiated goods j is produced according to the production function

$$Y_t(j) = Z_t N_t(j)^{1-\alpha} K_t(j)^{\alpha}, \tag{11}$$ Intermed

where the Z_t denotes a productivity shock, whose unconditional expectation equals $Z = 1$. Off the balanced growth path the log of Z_t is governed by

$$\ln Z_t = \rho_Z \ln Z_{t-1} + \epsilon_t^Z, \tag{12}$$ A_Shock

where ϵ_t^Z is normally distributed with mean zero and standard deviation σ_Z.

Without any nominal frictions each intermediate goods producer chooses his optimal relative price $P_t(j)/P_t$. In the symmetric equilibrium the relative price of all producers equals unity, individual output $Y_t(j)$ equals aggregate production Y_t and the equilibrium in the markets for labor and capital services implies

FME1

$$w_t = g_t(1-\alpha) Z_t N_t^{-\alpha} K_t^{\alpha}, \tag{13a}$$ FME1a

$$r_t = g_t \alpha Z_t N_t^{1-\alpha} K_t^{\alpha-1}, \tag{13b}$$ FME1b

$$g_t = \frac{\epsilon - 1}{\epsilon} \; \forall t,$$

where aggregate labor and capital equal

$$N = \sum_{s=1}^{R-1} \sum_{h=1}^{m} \psi_s \nu_h e_s z_h n, \tag{14}$$ N

and

$$K_t = \sum_{s=2}^{T} \sum_{h=1}^{m} \psi_s \nu_h k_{t,s,h}, \tag{15}$$ K

respectively. g_t denotes marginal costs. Aggregate profits in the intermediate goods sector, thus, amount to

$$\Omega_t = (1-g) Y_t, \tag{16}$$ A_Omega

where

$$Y_t = Z_t N^{1-\alpha} K_t^{\alpha}. \tag{17}$$ Y

3.3 Government

The government uses its tax income

$$Tax_t = \sum_{s=1}^{R-1}\sum_{h=1}^{m}\psi_s\nu_h\tau e_s z_h w_t n + \sum_{s=2}^{T}\sum_{h=1}^{m}\psi_s\nu_h\tau(r_t-\delta)k_{t,s,h} + \sum_{s=1}^{T}\sum_{h=1}^{m}\psi_s\nu_h\tau\omega_{t,s,h} \quad (18)$$ Tax1

and aggregate bequests Beq_t to finance pensions,

$$Pens = \sum_{s=R}^{T}\sum_{h=1}^{m}\psi_s\nu_h pens_h, \quad (19)$$ Pens

the purchase of goods G_t, and distributes any remaining funds lump sum to households. Thus, aggregate transfers (which equal individual transfers due to the unit mass of households) are given by

$$Trs_t = Tax_t + Beq_t - Pens - G_t. \quad (20)$$ Trs

Note that given the factor market equilibrium conditions (13) and the definitions of N and K_t in equations (14) and (15) tax income can be written as

$$Tax_t = \tau[Y_t - \delta K_t]. \quad (21)$$ Tax2

3.4 Aggregate Bequests

We are now in the position to derive a consistent definition of aggregate bequests from the flow budget constraints that we have specified so far. Aggregating over the budget constraints (3) of all households and taking into account the definitions in equations (4), (18), (21), (19), (13), (15), (14), yields

$$\begin{aligned} C_t &= \sum_{s=1}^{T}\sum_{h=1}^{m}\psi_s\nu_h c_{t,s,h}, \\ &= Y_t - G_t + (1-\delta)K_t + Beq_t - \sum_{s=1}^{T-1}\sum_{h=1}^{m}\psi_s\nu_h k_{t+1,s+1,h}. \end{aligned} \quad (22)$$ SumBC1

In order to add up to the resource constraint

$$C_t = Y_t + (1-\delta)K_t - G_t - K_{t+1}$$

the rightmost term on the last line of this equation must equal $Beq_t + K_{t+1}$. Since K_{t+1} can be written as

$$K_{t+1} = \sum_{s=1}^{T-1}\sum_{h=1}^{m}\phi_s\psi_s k_{t+1,s+1,h},$$

this implies:

$$Beq_t = \sum_{s=1}^{T-1} \sum_{h=1}^{m} (1-\phi_s)\psi_s \nu_h k_{t+1,s+1,h}. \tag{23}$$

A_Bequests

PFA

3.5 Perfect Annuity Markets

As an alternative to the treatment of bequests in the previous section assume that each generation s can insure against sudden death in period t. While alive, a member of generation s receives insurance payments of $(1-\phi_s)k_{t+1,s+1,h}$. Should he die at the end of age s, the insurance company receives his assets $k_{t+1,s+1,h}$. Given perfect insurance markets, aggregate payments of insurance premia amount to

$$\sum_{s=1}^{T-1} \sum_{h=1}^{m} \psi_s \nu_h (1-\phi_s) k_{t+1,s+1,h},$$

which equals aggregate bequests as defined in equation (23) [A_Bequests]. In this case, the budged constraints of generations $s = 1, \dots, R-1$ are given by

$$\begin{aligned} c_{t,s,h} &= (1-\tau)e_s z_h w_t n + [1 + (1-\tau)(r_t - \delta)]k_{t,s,h} + (1-\tau)\omega_{t,s,h} + trs_t \\ &\quad + (1-\phi_s)k_{t+1,s+1,h} - k_{t+1,s+1,h}, \\ &= (1-\tau)e_s z_h w_t n + [1 + (1-\tau)(r_t - \delta)]k_{t,s,h} + (1-\tau)\omega_{t,s,h} + trs_t \\ &\quad - \phi_s k_{t+1,s+1,h}, \end{aligned} \tag{24}$$

BR2a

where $k_{t,1,s} = 0$. For generations $s = R, \dots, T-1$ the budget constraints are

$$c_{t,s,h} = pens_h + [1 + (1-\tau)(r_t - \delta)]k_{t,s,h} + (1-\tau)\omega_{s,h} + trs_t - \phi_s k_{t+1,s+1,h}. \tag{25}$$

BR2b

Since all members of generation T die with probability one, the budget constraint of these agents is

$$c_{t,T,h} = pens_h + [1 + (1-\tau)(r_t - \delta)]k_{t,T,h} + (1-\tau)\omega_{T,h} + trs_t. \tag{26}$$

BR2c

Aggregation of the budget constraints over all generations and all productivity types yields the economy's resource constraint,

$$C_t = Y_t + (1-\delta)K_t - G_t - K_{t+1},$$

since the terms $\phi_s k_{t+1,s+1,h}$ aggregate to K_{t+1}.

With perfect annuity markets the coefficients of the matrix $A = (a_{ij})$ in (7) must be changed to:

$$\begin{aligned} a_{s+1,s} &= -A, \quad A := [1 + (1-\tau)(r-\delta)], \\ a_{s,s} &= \phi_s + AB_s, \quad B_s := [\beta(\phi_{s+1}/\phi_s)(1 + (1-\tau)(r-\delta))]^{\frac{1}{\gamma(1-\sigma)-1}}, \\ a_{s,s+1} &= -\phi_{s+1}B_s. \end{aligned} \tag{27}$$

The left-hand side vector does not change.

3.6 Calibration

The calibration follows the one in Section 3 of the main paper. Table 1 summarizes our parameter choice for Model 1.

Table 1

Parameterization of Model 1

Demographics	T=240	R=161	σ_x=3.6	
Preferences	β=0.9975	σ=2	γ^0=0.9775	γ^1=0.9740
Production	α=0.36	δ=0.019	ρ_Z=0.95	σ_Z=0.007
Market Structure	ϵ=6.0			
Government	ζ=0.5	τ=0.104	$\xi=\{0,1\}$	

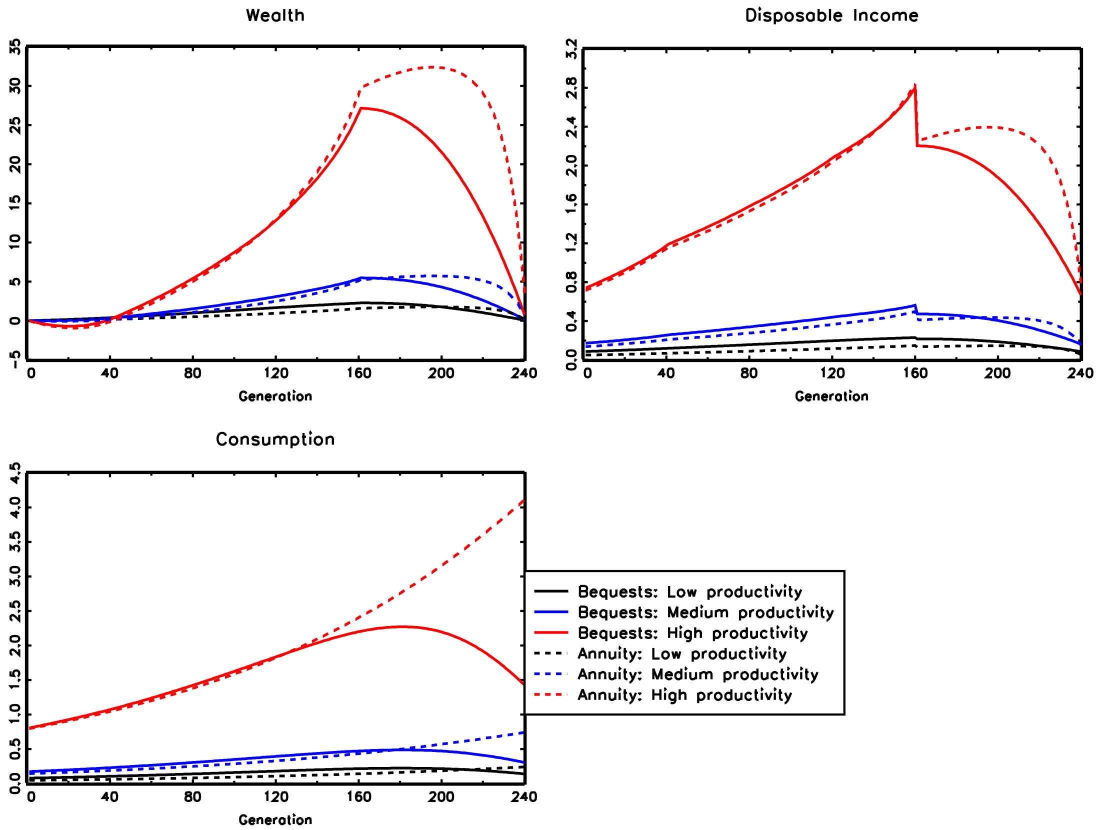
3.7 Stationary Equilibrium

We compute the stationary equilibrium in two steps. In the first step we assume that profits are distributed in equal shares to all households. Given initial values of N (from (14)), K, and Beq we can infer w and r from (13) (for $Z = 1$), $Pens$ from (19) and (5), Ω from (16), taxes from (21), and transfers from (20). We then solve the system (7) for the asset holdings of all agents. From this solution we compute K and Beq. The zero of this map in (K, Beq) provides the stationary solution of our model.

We use the asset shares $\kappa_{s,h}$ implied by this solution as initial values for a system of non-linear equations in $(K, Beq, \kappa_{s,h})$. The solution of this system in $2T + 2$ variables is the stationary solution of Model 1.

Figure 3 displays the age profiles of wealth, disposable income, and consumption in the case of $m = 3$ productivity groups, $m = 3$, and $\xi = 0$, for both the model with perfect

Figure 3: Stationary Equilibrium of Model 1



annuity markets and the model where the government confiscates bequests. The model with perfect annuity markets implies that consumption increases monotonically with age in almost the same manner as in a model with deterministic life time. Since the hump shaped consumption-age profile is more in accordance with empirical evidence, we restrict further attention to the latter model.

4 Endogenous Labor Supply and Progressive Taxation

In this section we add labor supply and progressive taxation of market income to the model of Section 3. We refer to this extended framework as Model 2.

CTS

4.1 The Tax Schedule

Gouveira and Stauss (1994) characterize the US effective income tax in the year 1989 with the function

$$\tau(y) = a_0 y - a_0 \left(y^{-a_1} + a_2\right)^{\frac{-1}{a_1}}, \tag{28}$$

TF0

and estimate its parameters as $a_0 = 0.258$, $a_1 = 0.786$, and $a_2 = 0.031$. We must adjust this function to our model, since we assume quarterly tax payments and since the units of income y in our model differ from those in the US. We assume that the average tax rate on an annual income equals the average tax rate on quarterly income, and that the average tax rate in our model equals the average tax rate on the average US income in 1989, y_{us}:

$$\frac{a_0(0.25y_{us}) - a_0\left((0.25y_{us})^{-a_1} + a_2\right)^{\frac{-1}{a_1}}}{(0.25y_{us})} = \frac{a_0 y - a_0\left(y^{-a_1} + \tilde{a}_2\right)^{\frac{-1}{a_1}}}{y}.$$

Solving this equation for $\tilde{a}_2$ yields our adjusted tax schedule:

$$\begin{aligned} \tau(y) &= a_0 y - a_0 \left(y^{-a_1} + \tilde{a}_2\right)^{\frac{-1}{a_1}}, \\ \tilde{a}_2 &= a_2 \left(4y_{us}/y\right)^{a_1}. \end{aligned} \tag{29}$$

A_TF1

Note, that the average income y in our model depends itself on the tax schedule. Therefore, we must adjust $\tilde{a}_2$ in each step of our iterative computation of the stationary equilibrium until convergence is achieved.

Given the tax function (A_TF1 29), aggregate taxes equal

$$Tax_t = \sum_{s=1}^{T} \sum_{h=1}^{m} \psi_s \nu_h \tau(y_{t,s,h}), \tag{30}$$

A_Tax3

where $y_{t,s,h}$ is the taxable income of household (s, h), which we define in the next subsection.

4.2 Households

We include leisure $1 - n_{t,s,h}$ additively separably in the instantaneous utility function of the households. At calendar time t an agent of age $s \in \{1, 2, \ldots, T\}$, who belongs to productivity group $h \in 1, 2, \ldots, m$, maximizes his expected life-time utility

$$U_{t,s,h} := \mathbb{E}_t \left[\sum_{j=s}^{T} \beta^{j-s} \prod_{i=s}^{j} \phi_i \left(\frac{c_{t_j,s,h}^{\gamma(1-\sigma)} - 1}{1-\sigma} + \eta_0 \frac{(1 - n_{t_j,s,h})^{1-\eta}}{1-\eta} \right) \right] \tag{31}$$

ltu1

subject to the sequence of budget constraints

$$\begin{aligned}
j &= s, s+1, \dots, T: \\
y_{t_j,s,h} &= e_j z_h w_{t_j} n_{t_j,s,h} + (r_{t_j} - \delta) k_{t_j,s,h} + \omega_{t_j,s,h}, \\
c_{t_j,s,h} &= y_{t_j,s,h} - \tau(y_{t_j,s,h}) + pens_{j,h} + trs_{t_j} + k_{t_j,s,h} - k_{t_j+1,s+1,h}, \\
e_j &= 0 \text{ for } j = R, R+1, \dots, T, \\
pens_{j,h} &= 0 \text{ for } j = 1, 2, \dots, R-1, \qquad (32) \\
k_{t_j,s,h} &= 0 \text{ for } j = 1, \\
k_{t_j+1,s+1,h} &= 0 \text{ for } j = T, \\
n_{t_j,s,h} &= 0 \text{ for } j = R, \dots, T, \\
t_j &:= t + j - s.
\end{aligned}$$

BC1

At time t the first-order conditions of all living agents of age s and type h consist of their respective budget constraint and the following $2T + R - 2$ equations: FOC2

$$\begin{aligned}
& s = 1, \dots, T: \\
& \lambda_{t,s,h} = \gamma c_{t,s,h}^{\gamma(1-\sigma)-1}, && (33a) \\
& s = 1, \dots, R-1: \\
& \eta_0 (1 - n_{t,s,h})^{-\eta} = \lambda_{t,s,h} (1 - \tau'(y_{t,s,h})) e_s z_h w_t, && (33b) \\
& s = 1, \dots, T-1: \\
& \lambda_{t,s,h} = \beta \phi_{s+1} \mathbb{E}_t \lambda_{t+1,s+1,h} (1 + (1 - \tau'(y_{t+1,s+1,h}))(r_{t+1} - \delta)), && (33c)
\end{aligned}$$

FOC2a FOC2b FOC2c

where $\lambda_{t,s,h}$ is the Lagrange multiplier attached to the budget constraint at time t for the s quarter old agent of productivity type h.

4.3 Calibration

We choose η_0 so that average working hours

$$\bar{H} := \sum_{s=1}^{R-1} \sum_{h=1}^{m} \frac{\psi_s}{\sum_{s=1}^{R-1} \psi_s} \nu_h n_{s,h} \tag{34}$$

AvH

in the model of Section 5 [Mod3] are 0.33. This requires two different setting, depending on $\xi \in \{0, 1\}$. The value of $\eta = 7$ implies a conservative estimate of the Frisch labor supply elasticity of 0.3. The remaining parameters are equal to those displayed in Table 1 [Pars1] and are summarized in Table 2 [Pars2].

Pars2

Table 2

Parameterization of Model 2

Demographics	T=240	R=161	σ_x^0=7.0	σ_x^1=4.2
Preferences	β=0.9975	σ=2	γ^0=0.9775	γ^1=0.9740
	η_0^0=0.06	η_0^1=0.90	η=7	
Production	α=0.36	δ=0.019	ρ_Z=0.95	σ_Z=0.007
Market Structure	ϵ=6.0			
Government	ζ=0.5	ξ={0, 1}		
	a_0=0.258	a_1=0.786	$\frac{\tau(Y-\delta K)}{Y-\delta K}$=0.104	

4.4 Stationary Equilibrium

Figure 4 displays the age profile of wealth, disposable income, consumption, and working hours. The broken lines correspond to the solution of model, if the tax schedule is linear with rate $\tau = 0.104$. This is the same tax rate that we used in Section 3.

The lower right panel of Figure 4 displays the age-profile of labor supply. Remember from Figure 2 that during the first 30 years of working life all types of agents face an upward sloping age-productivity profile. Despite this, the lower right panel of Figure 4 shows that the age-profile of labor supply is almost flat during the first 10 years and then starts to decline for all types of agents due to the negative wealth effect on labor supply. Since the low-productivity worker receive a large share of their income as government transfers, they supply less labor than the medium productivity workers (compare the black to the blue lines). The negative impact of the tax progression on labor supply declines with the agents' level of productivity. Nevertheless, low and medium productivity workers accumulate more wealth under the progressive tax system, whereas the high productivity workers save significantly less.

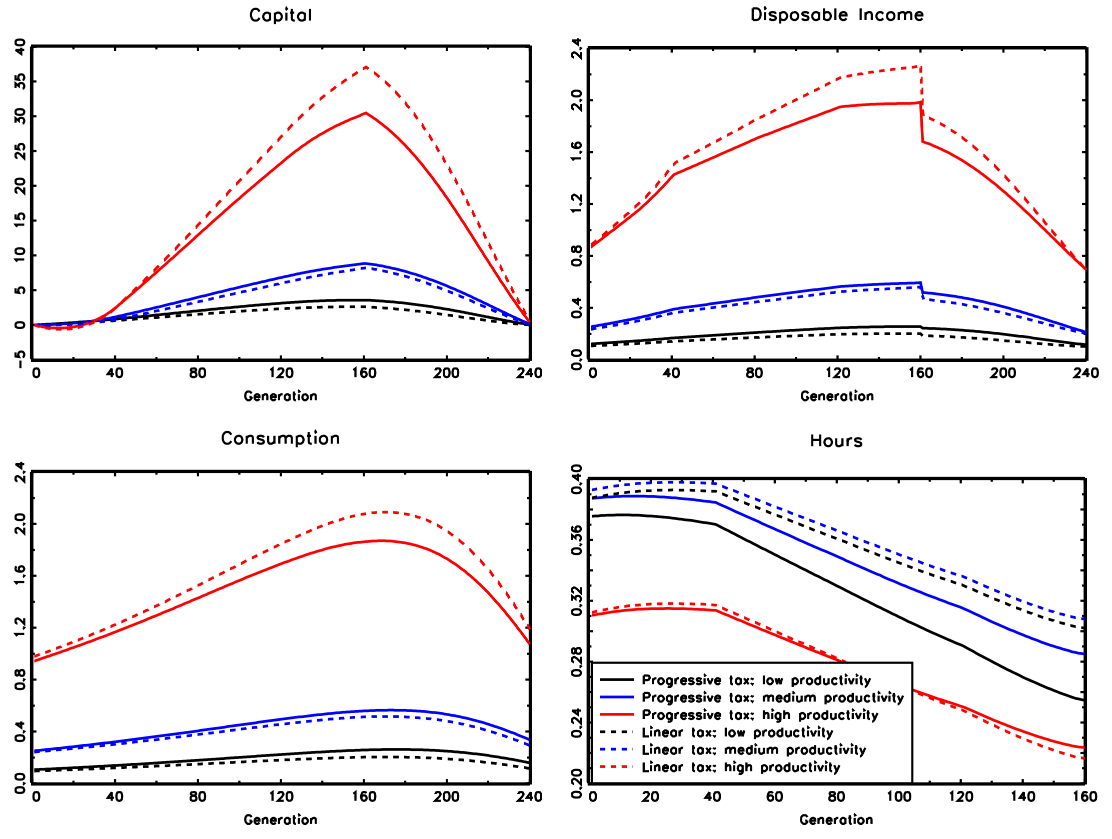
Wenn Du Graphik 6 ansiehst - dort bieten die mit niedriger Produktivität mehr an, wenn $Trs = 0$ ist - dann muss dieser Satz richtig sein. Fig7

LLMod2

4.5 Log-Linearization

To compute the model's business cycle dynamics we log-linearize it at the stationary solution as described in Heer and Maußner (2009a), Chapter 2. Our solution rests on

]
Figure 4: Fig4 Stationary Equilibrium of Model 2, $G_t = 0$



l11 the following linear model

$$C_u \mathbf{u}_t = C_{x\lambda} \begin{bmatrix} \mathbf{x}_t \\ \boldsymbol{\lambda}_t \end{bmatrix} + C_z \mathbf{z}_t, \tag{35a}$$ l11a

$$D_{x\lambda} \mathbb{E}_t \begin{bmatrix} \mathbf{x}_{t+1} \\ \boldsymbol{\lambda}_{t+1} \end{bmatrix} + F_{x\lambda} \begin{bmatrix} \mathbf{x}_t \\ \boldsymbol{\lambda}_t \end{bmatrix} = D_u \mathbb{E}_t \mathbf{u}_{t+1} + F_u \mathbf{u}_t + D_z \mathbb{E}_t \mathbf{z}_{t+1} + F_z \mathbf{z}_t, \tag{35b}$$ l11b

$$\mathbf{z}_{t+1} = \Pi \mathbf{z}_t + \boldsymbol{\epsilon}_{t+1}, \tag{35c}$$ l11c

$$\boldsymbol{\epsilon} \sim N(0, \Sigma).$$

The vector $\mathbf{x}_t$ consists of the (percentage deviations of) variables with given initial conditions. In our model, this is the $m(T-1)$-vector of capital stocks

$$\mathbf{x}_t := \left[\hat{k}_{t,2,1}, \dots, \hat{k}_{t,T,m}\right]'.$$

The vector $\boldsymbol{\lambda}_t$ summarizes variables which are also predetermined at time t but whose initial values must be chosen so as to satisfy the model's transversality conditions. The vector $\mathbf{u}_t$ holds all remaining variables of the model, which are determined given $\mathbf{x}_t$, $\boldsymbol{\lambda}_t$, and the vector of shocks $\mathbf{z}_t$.

Equations (35) can be reduced to

$$B\mathbb{E}_t \begin{bmatrix} \mathbf{x}_{t+1} \\ \boldsymbol{\lambda}_{t+1} \end{bmatrix} = A\mathbb{E}_t \begin{bmatrix} \mathbf{x}_t \\ \boldsymbol{\lambda}_t \end{bmatrix} + C\mathbf{z}_t. \tag{36}$$

In some versions of our model the matrix B is not invertible, since it is not always obvious which variables are costate rather than control variables, i.e., for which no static equations – as in (35a) – are available. Instead of doing tedious linear algebra to figure this out, we used the generalized Schur factorization to solve the system (36) for the linear policy functions

$$\mathbf{u}_t = L_x^u \mathbf{x}_t + L_z^u \mathbf{z}_t, \tag{37a}$$

$$\boldsymbol{\lambda}_t = L_x^\lambda \mathbf{x} + L_z^\lambda \mathbf{z}_t, \tag{37b}$$

$$\mathbf{x}_{t+1} = L_x^x \mathbf{x}_t + L_z^x \mathbf{z}_t, \tag{37c}$$

as explained in Heer and Maußner (2009b).

In the linearized version of Model 2 the vector $\mathbf{x}_t$ is composed of the percentage deviations of the individual capital stocks $\hat{k}_{t,s,h}$:

$$\mathbf{x}_t := \left[\hat{k}_{t,2,1}, \ldots, \hat{k}_{t,T,1}, \hat{k}_{t,2,2}, \ldots, k_{t,T,m}\right]'.$$

The variables in $\boldsymbol{\lambda}_t$ correspond to the percentage deviations of the $m(T-1)$ Lagrange multipliers of generations $s = 1$ through $s = T - 1$:

$$\boldsymbol{\lambda}_t := \left[\hat{\lambda}_{t,1,1}, \ldots, \hat{\lambda}_{t,T-1,m}\right]'.$$

The multipliers of generation $s = T$, $\lambda_{t,T,h}$, are determined via the budget constraint of this generation and, thus, are control rather than costate variables. The vector $\mathbf{u}_t$ is composed of the mT-vector of percentage deviations of consumption,

$$\hat{\mathbf{c}}_t := [\hat{c}_{t,1,1}, \ldots, \hat{c}_{t,T,m}]',$$

the m-vector

$$\left[\hat{\lambda}_{t,T,1}, \ldots, \hat{\lambda}_{t,T,m}\right]',$$

the $m(R-1)$-vector

$$\hat{\mathbf{n}}_t := [\hat{n}_{t,1,1}, \ldots, \hat{n}_{t,R-1,m},]',$$

the mT-vector

$$\hat{\mathbf{y}}_t := \left[\hat{y}_{t,1,1}, \dots, \hat{y}_{t,T,m}\right]',$$

the mT-vector of the percentage deviations of disposable income

$$\hat{\mathbf{y}}_t^d := \left[\hat{y}^d_{t,1,1}, \dots, \hat{y}^d_{t,T,m}\right],$$

where

$$y^d_{t,s,h} := \begin{cases} y_{t,s,y} - \tau(y_{t,s,h}) + trs_t & \text{for } s = 1, \dots, R-1 \\ y_{t,s,y} - \tau(y_{t,s,h}) + trs_t + Pens_h & \text{for } s = R, \dots, T. \end{cases} \tag{38}$$ yd

and the following variables:

$$\left[\hat{w}_t, \hat{r}_t, \hat{N}_t, \hat{H}_t, \hat{Y}_t, \hat{K}_t, \hat{C}_t, \hat{I}_t, \hat{\Omega}_t, \widehat{Tax}_t, \widehat{Trs}_t, \widehat{Beq}_t\right]'.$$

Aggregate working hours H_t are defined as

$$H_t := \sum_{s=1}^{R-1} \sum_{h=1}^{m} \psi_s \nu_h n_{t,s,h}. \tag{39}$$ AggHours

We first derive the set of equations (ll1a 35a). The log-linearized definitions of market

LL1 income $y_{t,s,h}$ are

$$h = 1, \dots, m, \tag{40a}$$ LL1a

$$s = 1, \dots, R-1:$$

$$y_{s,h}\hat{y}_{t,s,h} - e_s z_h w n_{s,h}\hat{w}_t - e_s z_h w n_{s,h}\hat{n}_{t,s,h} - \kappa_{s,h}\Omega\hat{\Omega}_t - rk_{s,h}\hat{r}_t = (r-\delta)k_{s,h}\hat{k}_{t,s,h},$$

$$s = R, \dots, T:$$

$$y_{s,h}\hat{y}_{t,s,h} - \kappa_{s,h}\hat{\Omega}_t - rk_{s,h}\hat{r}_t = (r-\delta)k_{s,h}\hat{k}_{t,s,h}.$$

The definitions of disposable income imply:[2]

$$h = 1, \dots, m, s = 1, \dots T: \tag{40b}$$ LL1b

$$y^d_{s,h}\hat{y}^d_{t,s,h} - (1-\tau'(y_{s,h}))y_{s,h}\hat{y}_{t,s,h} - trs\,\widehat{trs}_t = 0.$$

The log-linearized first-order conditions with respect to consumption and labor are:

$$h = 1, \dots, m,$$

$$s = 1, \dots, T: \tag{40c}$$

$$[\gamma(1-\sigma)-1]\hat{c}_{t,s,h} = \hat{\lambda}_{s,t,h},$$

$$s = 1, \dots, R-1: \tag{40d}$$ LL1c

$$\eta \frac{n_{s,h}}{1-n_{s,h}}\hat{n}_{t,s,h} - \hat{w}_t + \frac{\tau''(y_{s,h})}{1-\tau(y_{s,h})}y_{s,h}\hat{y}_{t,s,h} = \hat{\lambda}_{t,s,h}.$$

[2]Remember, individual pensions are independent of time.

The m log-linearized budget constraints of generation T yield:

$$
\begin{aligned}
&h = 1, \dots, m: \qquad (40e)\\
&c_{T,h}\hat{c}_{t,T,h} - y^d_{T,h}\hat{y}^d_{t,T,h} = k_{T,h}\hat{k}_{t,T,h}.
\end{aligned}
$$

The log-linear factor market equilibrium conditions are:

$$\hat{w}_t + \alpha\hat{N}_t - \alpha\hat{K}_t = \hat{Z}_t, \tag{40f}$$
LL1d

$$\hat{r}_t + (\alpha - 1)\hat{N}_t + (1-\alpha)\hat{K}_t = \hat{Z}_t. \tag{40g}$$
LL1e

The log-linearized aggregate production function (17) is

$$\hat{Y}_t + (\alpha - 1)\hat{N}_t - \alpha\hat{K}_t = \hat{Z}_t. \tag{40h}$$
LL1p

The log-linearized definitions of aggregate labor input N_t, aggregate hours H_t, the aggregate capital stock K_t, aggregate consumption C_t, and aggregate tax income Tax_t are:

$$0 = N\hat{N}_t - \sum_{s=1}^{R-1}\sum_{h=1}^{m}\psi_s\nu_h e_s z_h n_{s,h}\hat{n}_{t,s,h}, \tag{40i}$$

$$0 = H\hat{H}_t - \sum_{s=1}^{R-1}\sum_{h=1}^{m}\psi_s\nu_h n_{s,h}\hat{n}_{t,s,h}, \tag{40j}$$

$$K\hat{K}_t = \sum_{s=2}^{T}\sum_{h=1}^{m}\psi_s\nu_h k_{s,h}\hat{k}_{t,s,h}, \tag{40k}$$

$$0 = C\hat{C}_t - \sum_{s=1}^{T}\sum_{h=1}^{m}\psi_s\nu_h c_{s,h}\hat{c}_{t,s,h}, \tag{40l}$$
LL1l

$$0 = Tax\widehat{Tax}_t - \sum_{s=1}^{T}\sum_{h=1}^{m}\psi_s\nu_h\tau'(y_{s,h})y_{s,h}\hat{y}_{t,s,h}. \tag{40m}$$

The log-linearized definition of aggregate profits (16) is

$$\hat{\Omega}_t - \hat{Y}_t = 0. \tag{40n}$$
LL1n

The log-linearized aggregate resource constraint $Y_t = C_t + I_t + G_t$ yields an equation for the percentage deviation of aggregate investment I_t from its steady state value $I = \delta K$:

$$0 = I\hat{I}_t + C\hat{C}_t + G\hat{G}_t - Y\hat{Y}_t, \tag{40o}$$
LL1f

where

$$G\hat{G}_t = \xi Tax\,\widehat{Tax}_t + \xi Beq\,\widehat{Beq}_t \tag{40p}$$

follows from our definition that aggregate government spending is equal to a fraction ξ of government income (taxes and bequests) minus pensions $Pens$. Since pensions are independent of time and $Trs_t = (1-\xi)(Tax_t + Beq_t - Pens)$,

$$Trs\,\widehat{Trs}_t - (1-\xi)Tax\,\widehat{Tax}_t = (1-\xi)Beq\,\widehat{Beq}_t. \tag{40q}$$

Log-linearizing (23) and substituting for $\hat{k}_{t+1,s+1,h}$ from the household's budget constraints yields an equation that determines the percentage change of bequests:

$$Beqs\widehat{Beq}_t = \sum_{s=1}^{T-1}\sum_{h=1}^{m}(1-\phi_s)\psi_s\nu_h\left[k_{s,h}\hat{k}_{t,s,h} + y^d_{s,h}\hat{y}^d_{t,s,h} - c_{s,h}\hat{c}_{t,s,h}\right]. \tag{40r}$$

The dynamic equations (35b) result from the log-linearized $m(T-1)$ budget constraints (32) and the $m(T-1)$ Euler equations (6b). Using the definition of disposable income from (38) the former are:

$$\begin{aligned}
&h = 1,\dots,m:\\
&s = 1: && (41a)\\
&\quad k_{s+1,h}\hat{k}_{t+1,s+1,h} = y^d_{s,h}\hat{y}^d_{t,s,h} - c_{s,h}\hat{c}_{t,s,h},\\
&s = 2,\dots,T-1: && (41b)\\
&\quad k_{s+1,h}\hat{k}_{t+1,s+1,h} - k_{s,h}\hat{k}_{t,s,h} = y^d_{s,h}\hat{y}^d_{t,s,h} - c_{s,h}\hat{c}_{t,s,h},
\end{aligned}$$

and the latter are given by

$$\begin{aligned}
&h = 1,\dots,m:\\
&s = 1,\dots,T-1:\\
&\quad \mathbb{E}_t\hat{\lambda}_{t+1,s+1,h} - \hat{\lambda}_{t,s,h} = -\beta\frac{\phi_{s+1}\lambda_{s+1,h}}{\lambda_{s,h}}(1-\tau'(y_{s+1,h}))r\mathbb{E}_t\hat{r}_{t+1} && (41c)\\
&\qquad\qquad + \beta\frac{\phi_{s+1}\lambda_{s+1,h}}{\lambda_{s,h}}(r-\delta)\tau''(y_{s+1,h})y_{s+1,h}\mathbb{E}_t\hat{y}_{t+1,s+1,h}.
\end{aligned}$$

4.6 A Representative Agent Version of Model 2

If we want to understand in which way the overlapping generations structure changes the business cycle dynamics of the economy we must separate its influence from other features of our model. For this reason we also consider a representative agent version of the model in this subsection.

The representative household maximizes

$$U_t := \mathbb{E}_t\left[\sum_{j=1}^{\infty}\beta^{j-t}\left(\frac{C_{t+j}^{\gamma(1-\sigma)}-1}{1-\sigma} + \eta_0\frac{(1-N_{t+j})^{1-\eta}}{1-\eta}\right)\right]$$

subject to his budget constraint

$$
\begin{aligned}
K_{t+1} + C_t &= Y_t - \tau(Y_t - \delta K_t) + Trs_t + (1-\delta)K_t, \\
Y_t &:= w_t N_t + r_t K_t + \Omega_t.
\end{aligned}
$$

The production sector is the same as in Section 3.2 so that the production function (17) and the related factor market equilibrium conditions (13) still apply.

The first-order conditions for the household's problem are

$$
\begin{aligned}
C_t^{\gamma(1-\sigma)-1} &= \lambda_t, && \text{(42a)} \\
\eta_0(1-N_t)^{-\eta} &= \lambda_t(1-\tau'(Y_t - \delta K_t))w_t, && \text{(42b)} \\
\lambda_t &= \beta \mathbb{E}_t \lambda_{t+1}\left(1 + (1-\tau'(Y_{t+1} - \delta K_{t+1}))(r_{t+1} - \delta)\right). && \text{(42c)}
\end{aligned}
$$

The log-linearized model is:

$$
\begin{aligned}
[\gamma(1-\sigma)-1]\hat{c}_t &= \hat{\lambda}_t, && \text{(43a)} \\
\frac{\eta N}{1-N}\hat{N}_t - \hat{w}_t + \frac{\tau''}{1-\tau'}Y\hat{Y}_t &= \frac{\tau''}{1-\tau'}\delta K\hat{K}_t + \hat{\lambda}_t, && \text{(43b)} \\
\alpha\hat{N}_t + \hat{w}_t &= \alpha\hat{K}_t + \hat{Z}_t, && \text{(43c)} \\
(\alpha-1)\hat{N}_t + \hat{r}_t &= (\alpha-1)\hat{K}_t + \hat{Z}_t, && \text{(43d)} \\
(\alpha-1)\hat{N}_t + \hat{Y}_t &= \alpha\hat{K}_t + \hat{Z}_t, && \text{(43e)} \\
\frac{C}{I}\hat{C}_t - \frac{Y}{I}\hat{Y}_t + \frac{G}{I}\hat{G}_t + \hat{I}_t &= 0, && \text{(43f)} \\
-\frac{\tau'}{\tau}Y\hat{Y}_t + \hat{G}_t &= -\frac{\tau'}{\tau}\delta K\hat{K}_t, && \text{(43g)} \\
\delta\hat{I}_t &= \hat{K}_{t+1} + (\delta-1)\hat{K}_t, && \text{(43h)} \\
-\beta(1-\tau')r\mathbb{E}_t\hat{r}_{t+1} + \beta\tau''YE_t(r-\delta)\hat{Y}_{t+1} &= \beta\tau''(r-\delta)\delta K\hat{K}_{t+1} + \mathbb{E}_t\hat{\lambda}_{t+1} - \hat{\lambda}_t. && \text{(43i)}
\end{aligned}
$$

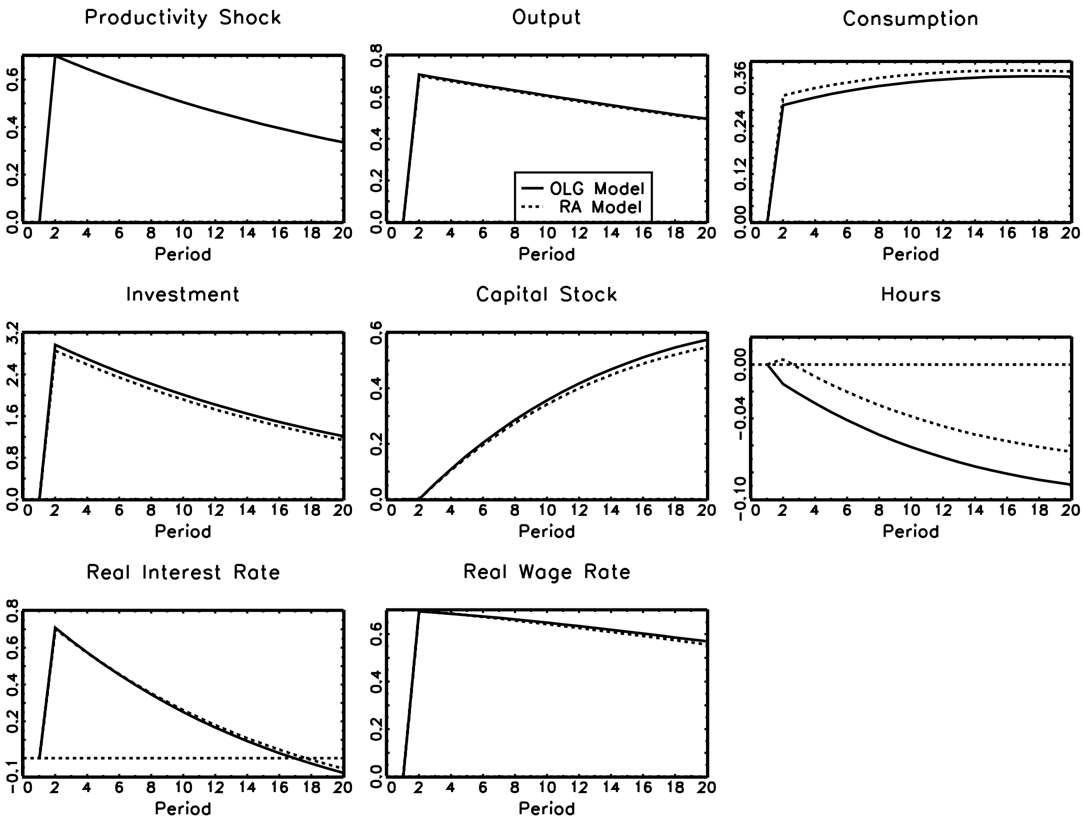
Equations (43a), (43b), and (43i) are the log-linearized first-order conditions (42), equations (43c) and (43d) are the log-linearized factor market equilibrium conditions (13), equation (43e) is the log-linearized production function (17), equation (43f) is the log-linearized resource constraint $Y_t = C_t + G_t + I_t$. The corresponding log-linearized definition of investment $I_t = K_{t+1} - (1-\delta)K_t$ is presented in (43h). Equation (43g) is the log-linearized definition of government expenditures on goods, $G_t = \xi(\tau(Y_t - \delta K_t)$.

4.7 Impulse Response

Figure 5 displays the response of both the OLG and the representative agent (RA) model to a technology shock in period $t = 2$. The most obvious difference between the two models concerns the response of hours, which is positive in the RA model and negative in the OLG model. In the latter model the increase in transfer payments creates a strong negative income effect on the low productivity workers which outweighs the positive substitution effect of higher wages.

]

Figure 5: Impulse Response in Model 2 Mod2



Mod3 5 Money in the Utility Function

5.1 Households

To motivate the holdings of money in our model we include the stock of real money balances in the current period utility functions of households. We refer to this extended model as Model 3. Let $x_{t,s,h}$ denote the stock of nominal money balances owned by the s year old household of productivity type h. For convenience, we set up the model in terms of the real value of the beginning-of-period money balances $m_{t,s,h} := x_{t,s,h}/P_{t-1}$.

$\pi_t := P_t/P_{t-1}$ denotes the inflation factor between the previous and the current quarter t.

At period t the expected life-time utility of household (s,h) is:

$$U_{t,s,h} := \mathbb{E}_t \left[\sum_{j=s}^{T} \beta^{j-s} \prod_{i=s}^{j} \phi_i \left(\frac{c_{t_j,s,h}^{\gamma(1-\sigma)} \left(\frac{m_{t_j,s,h}}{\pi_{t_j}}\right)^{(1-\gamma)(1-\sigma)} - 1}{1-\sigma} + \eta_0 \frac{(1-n_{t_j,s,h})^{1-\eta}}{1-\eta} \right) \right] \tag{44}$$

ltu2

To make this definition meaningful for the newborn agents without wealth, we assume that these agents receive a cash transfer from the government that equals the real beginning-of-period money balances of the one quarter older agents.[3]

In the sequence of budget constraints (BC1 32) we must change the definition of consumption to include the real, end-of-period money balances $m_{t,j,h}/\pi_t$ and the next-period real value of wealth held in terms of money $m_{t+1,j+1,h}$. The other equations are still valid:[4]

$$\begin{aligned} j &= s, s+1, \dots, T: \\ y_{t_j,s,h} &= e_j z_h w_{t_j} n_{t_j,s,h} + (r_{t_j} - \delta) k_{t_j,s,h} + \omega_{t_j,s,h}, \\ c_{t_j,s,h} &= y_{t_j,s,h} - \tau(y_{t_j,s,h}) + pens_{j,h} + trs_{t_j} + k_{t_j,s,h} + \frac{m_{t_j,s,h}}{\pi_{t_j}} \\ &\quad - k_{t_j+1,s+1,h} - m_{t_j+1,s+1,h}, \\ e_j &= 0 \text{ for } j = R, R+1, \dots, T, \\ pens_{j,h} &= 0 \text{ for } j = 1, 2, \dots, R-1, \\ k_{t_j,s,h} &= 0 \text{ for } j = 1, \\ k_{t_j+1,s+1,h} &= 0 \text{ for } j = T, \\ m_{t_j+1,s+1,h} &= 0 \text{ for } j = T, \\ n_{t,s,h} &= 0 \text{ for } j = R, \dots, T, \\ t_j &:= t + j - s. \end{aligned} \tag{45}$$

BC2

At time t the first-order conditions for maximizing (ltu2 44) subject to (BC2 45) of all living agents of type h consist of their respective budget constraints and the following $3(T-$

[3] In a previous version of the model we assumed that the cash transfer equals 21 percent of the average disposable income of the first generation, a value that corresponds to the average cash holdings of the 21-year-old households in the 1994 PSID survey. However, this value is far below the money holdings chosen by the one quarter older agents in our model, implying a huge, empirically implausible jump between the money holdings at $s=1$ and at $s=2$.

[4] For the newborn generation $j=1$ we assume that they receive $m_{t,1,h}$ as money transfer from the government so that $m_{t,1,h}$ rather than $m_{t,s,h}/\pi_t$ appears on the rhs of the budget constraint of this generation.

FOC7 1) + R equations:[5]

$$s = 1, \dots, T:$$

$$\lambda_{t,s,h} = \gamma c_{t,s,h}^{\gamma(1-\sigma)-1} \left(\frac{m_{t,s,h}}{\pi_t}\right)^{(1-\gamma)(1-\sigma)}, \tag{46a}$$ FOC7a

$$s = 1, \dots, R-1:$$

$$\eta_0(1-n_{t,s,h})^{-\eta} = \lambda_{t,s,h}(1-\tau'(y_{t,s,h}))e_s z_h w_t, \tag{46b}$$ FOC7b

$$s = 1, \dots, T-1:$$

$$\lambda_{t,s,h} = \beta\phi_{s+1}\mathbb{E}_t\lambda_{t+1,s+1,h}(1+(1-\tau'(y_{t+1,s+1,h}))(r_{t+1}-\delta)), \tag{46c}$$ FOC7c

$$\lambda_{t,s,h} = \beta\phi_{s+1}\mathbb{E}_t\lambda_{t+1,s+1,h}\left(\frac{1}{\pi_{t+1}} + \frac{1-\gamma}{\gamma}\frac{c_{t+1,s+1,h}}{m_{t+1,s+1,h}}\right). \tag{46d}$$ FOC7d

5.2 Monetary Authority

The aggregate stock of nominal money balances held by households of ages $s = 2$ through T equals

$$X_t := \sum_{s=2}^{T}\sum_{h=1}^{m}\psi_s\nu_h x_{t,s,h}. \tag{47}$$ AggM

The monetary authority imperfectly monitors the growth rate $(\theta_t - 1)$ of this aggregate and transfers the seignorage $Seign_t$ to the government. Thus:

$$X_{t+1} = \theta_t X_t, \tag{48}$$ A_MGrowth

$$Seign_t = \frac{X_{t+1}-X_t}{P_t} = (\theta_t - 1)\frac{X_t}{P_t} =: (\theta_t - 1)\frac{M_t}{\pi_t} \tag{49}$$ A_Seig

so that

$$M_t := \sum_{s=2}^{T}\sum_{h=1}^{m}\psi_s\nu_h m_{t,s,h}. \tag{50}$$ Mt

The percentage deviation of θ_t from its non-stochastic mean θ follows

$$\hat{\theta}_t = \rho_\theta\hat{\theta}_{t-1} + \epsilon_t^\theta, \quad \epsilon_t^\theta \sim N(0, \sigma_\theta^2). \tag{51}$$ Growthfactor

Our definition of M_t together with the assumptions regarding monetary policy imply a dynamic equation in the aggregate real beginning-of period money stock:

$$M_{t+1} = \frac{\theta_t M_t}{\pi_t}. \tag{52}$$ A_Mt2

[5] We simplified the first-order conditions with respect to $m_{t+1,s+1,h}$ by using (46a) FOC7a. This delivered equation (46d) FOC7d.

5.3 Government

Real government expenditures in period t consists of pensions $Pens_t$, the money endowment of the first generation,

$$\sum_{h=1}^{m} \psi_1 \nu_h m_{t,1,h}$$

government consumption G_t, and lump-sum transfers Trs_t to households. They are financed by the income tax defined in equation (30), confiscated bequests Beq_t, and seignorage:

$$G_t + Trs_t + Pens_t + \sum_{h=1}^{m} \psi_1 \nu_h m_{t,1,h} = Tax_t + Beq_t + (\theta_t - 1)\frac{M_t}{\pi_t}. \tag{53}$$

A_gbudget

To derive the economy's resource constraint from aggregation over the budget constraints of the households, the definition of bequests in (23) must be changed to include the wealth stored in real money holdings:

$$Beq_t = \sum_{s=1}^{T-1} \sum_{h=1}^{m} (1-\phi_s)\psi_s \nu_h \left(k_{t+1,s+1,h} + m_{t+1,s+1,h}\right). \tag{54}$$

Bequests2

We continue to assume that government consumption of goods is a fraction $\xi \in [0,1]$ of government income minus payments for pensions and money transfers to generation $s = 1$:

$$G_t = \xi \left(Tax_t + Beq_t + Seign_t - Pens - \sum_{h=1}^{m} \psi_1 \nu_h m_{t,1,h} \right). \tag{55}$$

GovExp

The model is closed by adding the production sector described in Section 3.2.

CalMod3 5.4 Calibration

In the stationary equilibrium of the deterministic version of Model 3 the inflation factor π equals the non-stochastic mean of the quarterly growth rate θ. We employ $\theta = 1.013$ from Cooley and Hansen (1995). We also take the estimates of the AR(1)-process (51) from these authors: $\rho_\theta = 0.49$ and $\sigma_\theta = 0.0089$. We choose the parameter γ so that the annualized average velocity of money PY/X in stationary equilibrium of our model equals the average velocity of M1 during 1960-2002 in the US of 6. Since this result depends upon our choice of $\xi \in \{0,1\}$, we introduce a superscript index from $\{0,1\}$ for the parameter γ, i.e. γ^0 and γ^1. The value of $\beta = 0.9975$ implies an annual capital-output-ratio of about 2.0, which is almost independent of our choice of ξ. The productivity of the two types of agents is chosen so that the Gini ratio of market income is 0.55. This implies $\sigma_x = 3.6$ irrespective of the value of ξ. The value of $\delta = 0.019$ implies an annual investment-capital ratio of 0.076 and is taken from Cooley and Prescott (1995). $\eta_0^0 = 0.6$ implies that average working hours $\bar{H}$ as defined in (34) is about 1/3 in the case of $\xi = 0$. If the excess of government income over pension payments is spent on goods, $\xi = 1$, the much smaller value of $\eta_0^1 = 0.19$ implies the same $\bar{H}$. The parameters of the tax schedule are chosen as explained in Section 4.1. The parameters of the AR(1)-process of the technology shock (12) are set equal to $\rho_Z = 0.95$ and $\sigma_Z = 0.007$ following Cooley and Prescott (1995). Table 3 summarizes our parameter choice for Model 3.

Table 3

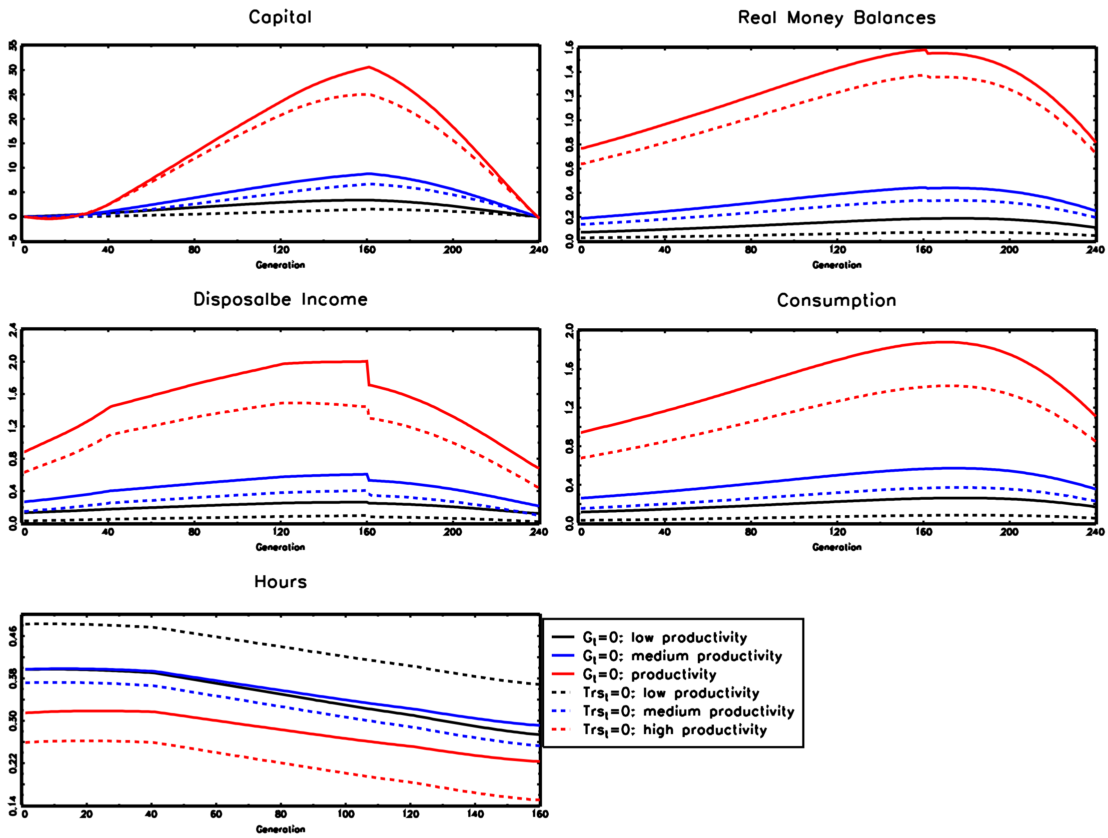
Parameterization of Model 3

Demographics	T=240	R=161	σ_x^0=3.6	
Preferences	β=0.9975	σ=2	γ^0=0.9775	γ^1=0.9740
	η_0^0=0.19	η_0^1=0.60	η=7	
Production	α=0.36	δ=0.019	ρ_Z=0.95	σ_Z=0.007
Market Structure	ϵ=6.0			
Government	ζ=0.5	$\xi=\{0,1\}$		
	a_0=0.258	a_1=0.786	$\frac{\tau(Y-\delta K)}{Y-\delta K}$=0.104	
	θ=1.013	ρ_M=0.49	σ_M=0.0089	

5.5 Stationary Equilibrium

Figure 6 displays the age profiles of capital, real money balances, disposable income, consumption and working hours. The effect of government transfers on individual labor supply is illustrated in the lower left panel. If the government spends its excess revenues on consumption, $Trs_t = 0$ there is a clear ranking of the labor-age profiles: the low productivity workers supply more working hours than the medium productivity workers, who in turn work more than the high productivity households. Due to the negative wealth effect, all three profiles decline at age of 30. However, if the government transfers its excess revenues lump-sum to the households, the low productivity workers decrease their labor supply relatively more than the medium productivity workers so that their age profile drops below the profile of the medium productivity workers (see the solid lines).

Figure 6: Stationary Equilibrium of Model 3



The discontinuous drop in money balances at age $s = 161$ can be understood by combining the the first-order conditions (46c) and (46d). Evaluated at the stationary equilibrium and rearranged, this gives:

$$\frac{m_{s+1,h}}{\theta} = \Big[\theta - 1 + \theta\,(1 - \tau'(y_{s+1,h})(r - \delta))\Big]^{-1}\left[\frac{1-\gamma}{\gamma}\right]c_{s+1,h}$$

At $s = R$ the decline in taxable income causes a sudden decrease in the marginal tax rate $\tau'(y_{s+1,h})$, which in turn explains the non-monotonic behavior of the money-age profile.

5.6 Log-Linearization

The log-linear version of the model set out in the previous subsection is a straight forward extension of the log-linear model considered in Subsection 4.5. We include the percentage deviations of beginning-of-period real money holdings in the vector of state variables $\mathbf{x}_t$:

$$\mathbf{x}_t := \left[\hat{k}_{t,2,1}, \ldots, \hat{k}_{t,T,m}, \hat{m}_{t,2,1}, \ldots, \hat{m}_{t,T,m}\right]'.$$

We assume that the money transfer to the first generation is kept fixed at its stationary value so that $\hat{m}_{t,1,h} = 0$ for all $h = 1, \dots, m$. We add $\hat{\pi}_t$ – the percentage deviation of the inflation factor from its stationary value θ – to the vector $\boldsymbol{\lambda}_t$:

$$\boldsymbol{\lambda}_t := \left[\hat{\lambda}_{t,1,1}, \dots, \hat{\lambda}_{t,T-1,m}, \hat{\pi}_t\right]',$$

and include the percentage deviation of aggregate beginning-of-period real money balances M_t from its stationary value in the vector $\mathbf{u}_t$ which is otherwise unchanged. The vector $\mathbf{z}_t$ now includes the percentage deviation of the level of productivity Z_t from its stationary value and the percentage deviation of the growth factor of money supply θ_t:

$$\mathbf{z}_t := \left[\hat{Z}_t, \hat{\theta}_t\right].$$

From the model of Subsection 4.5 the log-linear definitions of market income (40a) and disposable income (40b) still apply.

LL7 The log-linearized first-order conditions with respect to consumption are

$$\begin{aligned}
&h = 1, \dots, m,\\
&s = 1:\\
&\quad [\gamma(1-\sigma) - 1]\hat{c}_{t,s,h} = \hat{\lambda}_{t,s,h} + (1-\sigma)(1-\gamma)\hat{\pi}_t,\\
&s = 2, \dots, T-1:\\
&\quad [\gamma(1-\sigma) - 1]\hat{c}_{t,s,h} = \hat{\lambda}_{t,s,h} - (1-\gamma)(1-\sigma)\hat{m}_{t,s,h} + (1-\gamma)(1-\sigma)\hat{\pi}_t.
\end{aligned} \tag{56a}$$ LL7a

The log-linearized first-order conditions with respect to labor supply from (40d) remain valid. The m log-linearized budget constraints of generation T yield:

$$\begin{aligned}
&h = 1, \dots, m:\\
&c_{T,h}\hat{c}_{t,T,h} - y^d_{T,h}\hat{y}^d_{t,T,h} = k_{T,h}\hat{k}_{t,T,h} + \frac{m_{T,h}}{\theta}\hat{m}_{t,T,h} - \frac{m_{T,h}}{\theta}\hat{\pi}_t.
\end{aligned} \tag{56b}$$ LL7b

Also the log-linear equations (40f) through (40h) are unchanged. The log-linearized definition of aggregate money balances (50) is

$$M\hat{M}_t = \sum_{s=2}^{T}\sum_{h=1}^{m} \psi_s \nu_h m_{s,h}\hat{m}_{t,s,h}. \tag{56c}$$ LL7c

The definition of government expenditures (55) implies:

$$G\hat{G}_t = \xi\, Tax\,\widehat{Tax}_t + \xi\, Beq\,\widehat{Beq}_t + \xi(\theta-1)\frac{M}{\theta}\hat{M}_t - \xi(\theta-1)\frac{M}{\theta}\hat{\pi}_t + \xi M\hat{\theta}_t, \tag{56d}$$

where we assume that the money transfers to the first generation $m_{t,1,h}$ are kept at their stationary values. Analogously, the linearized equation for aggregate transfers is:

$$\begin{aligned} Trs\,\widehat{Trs}_t = (1-\xi)\,Tax\,\widehat{Tax}_t + (1-\xi)\,Beq\,\widehat{Beq}_t + (1-\xi)(\theta-1)\frac{M}{\theta}\hat{M}_t \\ -(1-\xi)(\theta-1)\frac{M}{\theta}\hat{\pi}_t + (1-\xi)M\hat{\theta}_t. \end{aligned} \tag{56e}$$

Finally, the log-linearized definition of bequests can be written as:

$$\begin{aligned} Beq\widehat{Beq}_t = \sum_{s=1}^{T-1}\sum_{h=1}^{m}(1-\phi_s)\psi_s\nu_h\Big[k_{s,h}\hat{k}_{t,s,h} + y^d_{s,h}\hat{y}^d_{t,s,h} - c_{s,h}\hat{c}_{t,s,h} \\ + \frac{m_{s,h}}{\theta}\hat{m}_{t,s,h} - \frac{m_{s,h}}{\theta}\hat{\pi}_t\Big]. \end{aligned} \tag{56f}$$

LL7f

LL8

The dynamic equations of the canonical model (35b) result from the log-linearized $m(T-1)$ budget constraints (45), the $2m(T-1)$ Euler equations (46c) and (46d), and (52). Using the definition of disposable income from (38) the former are:

$h = 1, \dots, m:$

$s = 1:$ (57a) LL8a

$$k_{s+1,h}\hat{k}_{t+1,s+1,h} + m_{s+1,h}\hat{m}_{t+1,s+1,h} = y^d_{s,h}\hat{y}^d_{t,s,h} - c_{s,h}\hat{c}_{t,s,h},$$

$s = 2, \dots, T-1:$ (57b) LL8b

$$\begin{aligned} &k_{s+1,h}\hat{k}_{t+1,s+1,h} + m_{s+1,h}\hat{m}_{t+1,s+1,h} \\ &- k_{s,h}\hat{k}_{t,s,h} - \frac{m_{s,h}}{\theta}\hat{m}_{t,s,h} + \frac{m_{s,h}}{\theta}\hat{\pi}_t = y^d_{s,h}\hat{y}^d_{t,s,h} - c_{s,h}\hat{c}_{t,s,h}. \end{aligned}$$

Among the latter, the log-linearized Euler equations (46c) are still given by (41c) whereas the log-linear version of (46d) is

$h = 1, \dots, m:$

$s = 1, \dots, T-1:$

$$\begin{aligned} &\mathbb{E}_t\hat{\lambda}_{t+1,s+1,h} - \hat{\lambda}_{t,s,h} - \beta\phi_{s+1}\frac{\lambda_{s+1,h}}{\theta\lambda_{s,h}}\mathbb{E}_t\hat{\pi}_{t+1} \\ &- \beta\phi_{s+1}\frac{\lambda_{s+1,h}}{\lambda_{s,h}}\frac{1-\gamma}{\gamma}\frac{c_{s+1,h}}{m_{s+1,h}}\hat{m}_{t+1,s+1,h} = -\beta\phi_{s+1}\frac{\lambda_{s+1,h}}{\lambda_{s,h}}\frac{1-\gamma}{\gamma}\frac{c_{s+1,h}}{m_{s+1,h}}\mathbb{E}_t\hat{c}_{t+1,s+1,h}. \end{aligned} \tag{57c}$$

Finally, the log-linearized equation (52) is:

$$\hat{M}_{t+1} - \hat{M}_t + \hat{\pi}_t = \hat{\theta}_t. \tag{57d}$$

LL8d

5.7 Results

Table 4 displays the results from simulations of Model 3 for the polar cases $\xi = 0$ and $\xi = 1$ and the progressive tax schedule (29). The numbers in parenthesis are simulation

Tab4

Table 4

Second Moments from Model 3

Variable	$\xi = 0$			$\xi = 1$		
	s_x	r_{xy}	r_x	s_x	r_{xy}	r_x
Output	0.91	1.00	0.70	0.94	1.00	0.70
	(0.91)	(1.00)	(0.70)	(0.94)	(1.00)	(0.70)
Consumption	0.39	0.98	0.72	0.31	0.96	0.73
	(0.38)	(0.98)	(0.72)	(0.30)	(0.97)	(0.73)
Investment	3.82	0.99	0.69	5.18	0.68	0.53
	(3.82)	(0.99)	(0.69)	(3.67)	(1.00)	(0.69)
Effective Labor	0.04	0.33	0.89	0.09	0.73	0.70
	(0.04)	(0.41)	(0.88)	(0.08)	(0.83)	(0.74)
Hours	0.04	−0.55	0.89	0.05	0.58	0.72
	(0.04)	(−0.49)	(0.93)	(0.05)	(0.72)	(0.78)
Real Wage	0.90	1.00	0.70	0.88	1.00	0.70
	(0.89)	(1.00)	(0.70)	(0.88)	(1.00)	(0.71)
Inflation	1.66	−0.10	−0.06	1.72	−0.04	−0.06
Gini Ratio						
Wealth	0.11	−0.13	0.92	0.07	−0.13	0.83
	(0.06)	(−0.01)	(0.95)	(0.04)	(0.01)	(0.96)
Market Income	0.02	−0.31	0.82	0.02	0.92	0.69
	(0.01)	(−0.95)	(0.70)	(0.02)	(0.96)	(0.69)
Disposable Income	0.65	−0.06	0.37	0.04	0.94	0.70
	(0.05)	(−0.97)	(0.74)	(0.04)	(0.96)	(0.70)

Notes: Second moments from HP-filtered simulated time series. The numbers are averages from 100 simulations. The length of each indivdual time series is 150 quarters. s_x:=standard deviation of variable x, r_{xy}:=cross correlation of variable x with output, r_x:=first order autocorrelation of variable x. Results from Model 2 in parenthesis.

results from Model 2. Adding money to the model of Section 4 has almost no impact on the volatility of aggregate output, investment, consumption, labor input, and the real wage. The rate of inflation is noncyclical and not persistent. The increased volatility of the distribution of wealth stems from the variation of individual money holdings. The drastic increase of the standard deviation of the Gini ratio of disposable income can be traced to the volatility of aggregate transfers, which in turn results from seignorage. Assuming a steady supply of money, $\sigma_M = 0$ reduces this ratio to about the value known from simulations of Model 2. Besides the obvious behavior of the Gini ratio of disposable income the most notable difference between $\xi = 0$ (government transfers) and $\xi = 1$ (government consumption) concerns the behavior of private consumption and investment. The volatility of transfers results in a higher standard deviation of private consumption, whereas the volatility of government consumption translates into a higher standard deviation of private investment.

5.8 The Representative Agent Version of Model 3

In this model the household solves

$$\max \quad U_t := \mathbb{E}_t \left\{ \sum_{j=1}^{\infty} \beta^{j-t} \left[\frac{C_{t+j}^{\gamma(1-\sigma)} \left(\frac{M_{t+j}}{P_{t+j}}\right)^{(1-\gamma)(1-\sigma)} - 1}{1-\sigma} + \eta_0 \frac{(1-N_{t+j})^{1-\eta}}{1-\eta} \right] \right\},$$

subject to

$$K_{t+1} + \frac{M_{t+1}}{P_t} + C_t = Y_t - \tau(Y_t - \delta K_t) + (1-\delta)K_t + \frac{M_t}{P_t} + Trs_t,$$
$$Y_t := w_t N_t + r_t K_t + \Omega_t = Z_t N_t^{1-\alpha} K_t^{\alpha}. \tag{58}$$

Model3

The government's budget constraint is

$$G_t + Trs_t = \tau(Y_t - \delta K_t) + (\theta_t - 1)\frac{M_t}{P_t},$$

where the rightmost term is seignorage. We continue to assume that government expenditures on goods are a fraction $\xi \in [0,1]$ of the government's revenues from taxes and from creating money:

$$G_t = \xi \left[\tau(Y_t - \delta K_t) + (\theta_t - 1)\frac{M_t}{P_t} \right]. \tag{59}$$

GovExpMod3

FOC4 The first-order conditions for the household's problem are:

$$\lambda_t = \gamma C_t^{\gamma(1-\sigma)-1} \left(\frac{M_t}{P_t}\right)^{(1-\gamma)(1-\sigma)}, \tag{60a}$$

FOC4a

$$\eta_0(1-N_t)^{-\eta} = \lambda_t(1 - \tau'(Y_t - \delta K_t))w_t, \tag{60b}$$

FOC4b

$$\lambda_t = \beta \mathbb{E}_t \lambda_{t+1} \left(1 + (1 - \tau'(Y_{t+1} - \delta K_{t+1}))(r_{t+1} - \delta)\right). \tag{60c}$$

FOC4c

$$\lambda_t = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{P_{t+1}/P_t} \left[1 + \frac{(1-\gamma)}{\gamma} \frac{C_{t+1}}{M_{t+1}/P_{t+1}} \right]. \tag{60d}$$

FOC4d

In the stationary equilibrium of the deterministic version of this model the stock of real money balances M/P is constant. Therefore, the growth factor of rate of money supply θ equals the growth factor of inflation $\pi_t := P_t/P_{t-1}$. Using $\lambda_t = \lambda_{t+1}$ in (60d) [FOC4d]

SS4 implies

$$\frac{M}{P} = \frac{\beta(1-\gamma)}{\gamma(\theta - \beta)} C. \tag{61a}$$

SS4a

In the stationary equilibrium the household's budget constraint reduces to $Y_t = C_t + G_t + \delta K_t$. Therefore, (61a) and (59) imply

$$C = \left(1 + \xi(\theta - 1)\frac{\beta(1-\gamma)}{\gamma(\theta-\beta)}\right)^{-1} \left[Y - \delta K - \xi\tau(Y - \delta K)\right]. \tag{61b}$$

SS4b

Together with the factor market equilibrium conditions in the stationary equilibrium

$$w = g(1-\alpha)N^{-\alpha}K^{\alpha}, \tag{61c}$$

$$r = g\alpha N^{1-\alpha}K^{\alpha-1}, \tag{61d}$$

the stationary versions of (60a)-(60c) and the equation of the tax schedule (28) imply two non-linear equations in N and K.

Log-linearized at the stationary equilibrium the dynamic model consists of two sets of

LL5a equations:

$$[\gamma(1-\sigma)-1]\hat{C}_t = -(1-\gamma)(1-\sigma)\hat{m}_t + (1-\gamma)(1-\sigma)\hat{\pi}_t + \hat{\lambda}_t, \tag{62a}$$

$$\frac{\eta N}{1-N}\hat{N}_t + \frac{\tau'' Y}{1-\tau'}\hat{Y}_t - \hat{w}_t = \frac{\tau'' I}{1-\tau'}\hat{K}_t + \hat{\lambda}_t, \tag{62b}$$

$$\alpha\hat{N}_t + \hat{w}_t = \alpha\hat{K}_t + \hat{Z}_t, \tag{62c}$$

$$(\alpha-1)\hat{N}_t + \hat{r}_t = (\alpha-1)\hat{K}_t + \hat{Z}_t, \tag{62d}$$

$$(\alpha-1)\hat{N}_t + \hat{Y}_t = \alpha\hat{K}_t + \hat{Z}_t, \tag{62e}$$

$$(C/I)\hat{C}_t - (Y/I)\hat{Y}_t + \hat{I}_t + (G/I)\hat{G}_t = 0, \tag{62f}$$

$$\frac{-\xi\tau' Y}{G}\hat{Y}_t + \hat{G}_t = -\frac{\xi\tau' I}{G}\hat{K}_t + \frac{\xi(\theta-1)(M/P)}{G}\hat{m}_t - \frac{\xi(\theta-1)(M/P)}{G}\hat{\pi}_t + \frac{\xi\theta(M/P)}{G}\hat{\theta}_t. \tag{62g}$$

LL5b and:

$$\hat{K}_{t+1} + (\delta-1)\hat{K} = \delta\hat{I}_t, \tag{63a}$$

$$\beta(r-\delta)\tau'' I\hat{K}_{t+1} + \mathbb{E}_t\hat{\lambda}_{t+1} - \hat{\lambda}_t = -\beta(1-\tau')rE_t\hat{r}_{t+1} + \beta\tau''(r-\delta)YE_t\hat{Y}_{t+1}, \tag{63b}$$

$$\mathbb{E}_t\hat{\lambda}_{t+1} - \frac{\beta}{\theta}\mathbb{E}_t\hat{\pi}_{t+1} + \frac{\beta-\theta}{\theta}\hat{m}_{t+1} - \hat{\lambda}_t = \frac{\beta-\theta}{\theta}\mathbb{E}_t\hat{C}_{t+1}, \tag{63c}$$

$$\hat{m}_{t+1} - \hat{m}_t + \hat{\pi}_t = \hat{\theta}_t, \tag{63d}$$

where $m_t := M_t/P_{t-1}$.

To match the canonical linear model (35), we put:

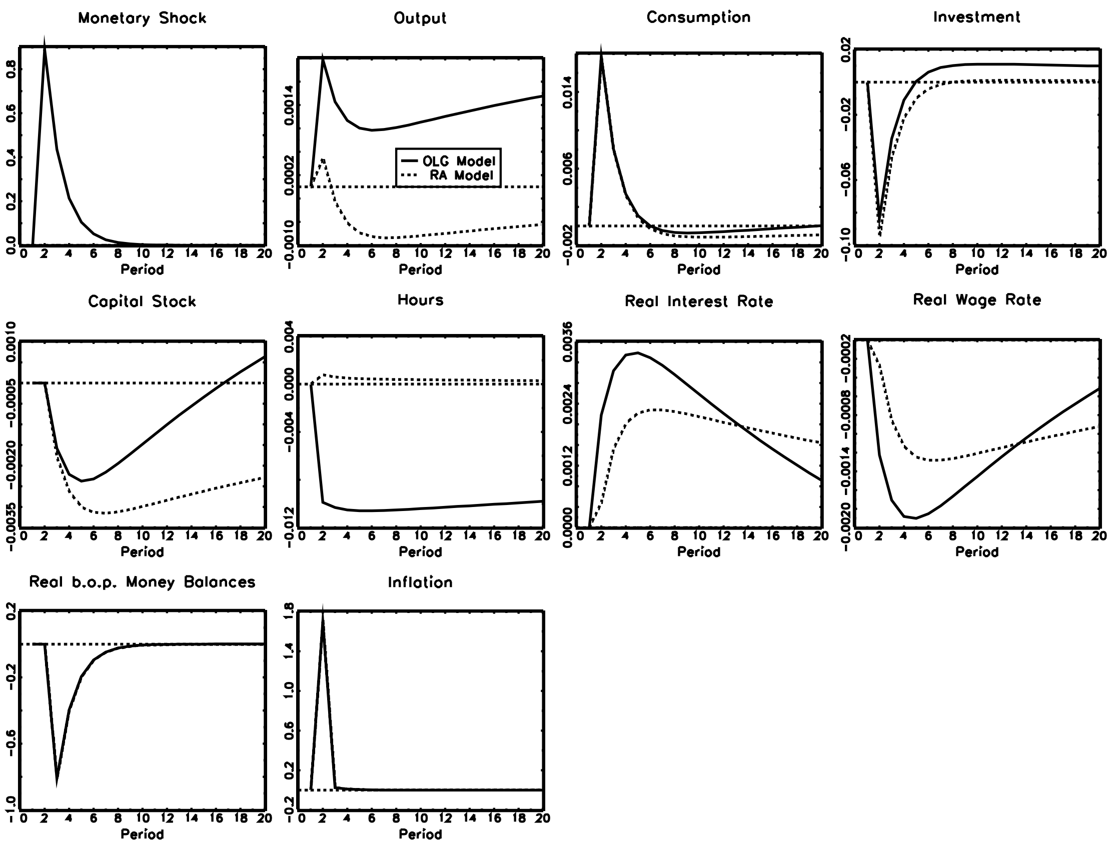
$$\mathbf{u}_t := \left[\hat{Y}_t, \hat{C}_t, \hat{I}_t, \hat{N}_t, \hat{w}_t, \hat{r}_t, \hat{G}_t\right]',$$

$$\mathbf{x}_t := \left[\hat{K}_t, \hat{m}_t\right]',$$

$$\boldsymbol{\lambda}_t := \left[\hat{\lambda}_t, \hat{\pi}_t\right]'.$$

Figure 7 displays the response of several variables to a monetary shock of size $\sigma_m = 0.0089$ in period $t = 2$ for both the OLG and the RA model. Note that the response is given in percentage points. The increase of the money growth factor by 0.89 percentage points triggers an increase in output of less than 0.0022 percentage points in the OLG model an of less than 0.0005 percentage points in the RA model. The negativ reaction of hours in the OLG model is, again, the result of the labor supply response of the low productivity workers, who decrease their labor supply in response to the increase of their transfer income. Since the high productivity workers increase their labor supply effective labor input increases and accounts for the positive effect of output.

Figure 7: Impulse Response to a Monetary Shock in Model 3



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6 Nominal Rigidities

In this section we introduce nominal rigidities in price setting and imperfect indexation of taxes and pensions into the model presented in the previous section. We refer to this extended model as Model 4.

6.1 Price Setting

Prices are set according to the mechanism spelled out in Calvo (1983). In each period a fraction of $(1-\varphi)$ of the firms in the intermediate goods sector are allowed to set their relative price P_{jt}/P_t optimally. Depending on the assumptions about the information set and the adjustment rule of the remaining fraction of firms we consider three different settings.

1. A purely forward-looking Phillips curve arises if the price setters choose their optimal relative price after the realization of the monetary shock and if the the other firms adjust their price according to

   $$P_{N,j,t} = \pi P_{N,j,t-1},$$

   where π denotes the stationary value of the inflation factor in the deterministic version of the model. In terms of percentage deviations from the stationary values the Phillips curve equation is:

   $$0 = \hat{\pi}_t - \beta \mathbb{E}_t \hat{\pi}_{t+1} - \frac{(1-\varphi)(1-\beta\varphi)}{\varphi}\hat{g}_t, \tag{64}$$

   A_PK0

   where $\hat{g}_t$ is the percentage deviation of marginal costs from its stationary value $g = (\epsilon - 1)/\epsilon$.

2. If we assume instead that non-optimizing firms set their price according to

   $$P_{N,j,t} = \pi_{t-1} P_{N,j,t-1}$$

   the log-linear Phillips curve becomes

   $$0 = \hat{\pi}_t - \frac{1}{1+\beta}\hat{\pi}_{t-1} - \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+1} - \frac{(1-\varphi)(1-\beta\varphi)}{(1+\beta)\varphi}\hat{g}_t. \tag{65}$$

   A_PK1

3. Finally, if the firms must set their optimal price before the realization of the money supply shock, equation (65) changes to

   $$0 = \hat{\pi}_t - \frac{1}{1+\beta}\hat{\pi}_{t-1} - \frac{\beta}{1+\beta}E_{t-1}\hat{\pi}_{t+1} - \frac{(1-\varphi)(1-\beta\varphi)}{(1+\beta)\varphi}E_{t-1}\hat{g}_t. \tag{66}$$

   A_PK2

6.2 Taxes and Pensions

We assume that taxes are collected from nominal income and are imperfectly indexed to inflation. Specifically, the tax schedule is adjusted to the actual price level with a lag of $n \in \{0, 1, \dots\}$ quarters where $n = 0$ is perfect indexation. In the intermediate quarters nominal income is deflated by $P_t^I := \pi^n P_{t-n}$. Accordingly, the tax schedule is now given by

$$\tau_{t,s,h} = \tau\left[\left(\prod_{i=0}^{n-1}\frac{\pi_{t-i}}{\pi}\right) y_{t,s,h}\right], \tag{67}$$

A_TF2

where $y_{t,s,h}$ is the market income in units of the final output (wage, capital, and profit income less depreciation of capital) as defined in (45).

Analogously, pension payments are given by

$$pens_{t,s,h} = \left(\prod_{i=0}^{n-1}\frac{\pi_{t-i}}{\pi}\right)^{-1} pens_h, \tag{68}$$

Pens1

with $pens_h$ defined in (5).

6.3 Temporary Equilibria

The imperfect indexation of taxes implies that the households' first-order conditions with respect to labor supply $n_{t,s,h}$ and next-period capital $k_{t+1,s+1,h}$ change from (46b) to

$$\eta_0(1 - n_{t,s,h})^{-\eta} = \lambda_{t,s,h}\left(1 - \tau'\left[\left(\prod_{i=0}^{n-1}\frac{\pi_{t-i}}{\pi}\right) y_{t,s,h}\right]\prod_{i=0}^{n-1}\frac{\pi_{t-i}}{\pi}\right) e_s z_h w_t, \tag{69a}$$

FOC8b

and from (46c) to

$$\lambda_{t,s,h} = \beta\phi_{s+1}\mathbb{E}_t\lambda_{t+1,s+1,h}\left(1 + \left(1 - \tau'\left[\left(\prod_{i=0}^{n-1}\frac{\pi_{t+1-i}}{\pi}\right) y_{t+1,s+1,h}\right]\prod_{i=0}^{n-1}\frac{\pi_{t+1-i}}{\pi}\right)(r_{t+1} - \delta)\right). \tag{69b}$$

FOC8c

Except for the definition of taxes and pensions the remaining equations of the model presented in Section 5 do not change. Therefore, our Model 4 has the same stationary equilibrium as Model 3.

6.4 Log-Linearization

The definitions of the vectors $\mathbf{x}_t$ and $\boldsymbol{\lambda}_t$ from the canonical linear model (35) depend on the Phillips curve equation. In the case of Phillips curves (64) and (65), we include $\hat{\pi}_t$ and $\hat{g}_t$ in the vector $\boldsymbol{\lambda}_t$, thus:

$$\boldsymbol{\lambda}_t := \left[\hat{\lambda}_{t,1,1}, \dots, \hat{\lambda}_{t,T-1,m}, \hat{\pi}_t, \hat{g}_t\right]'.$$

When we use equation (66) so that $\hat{\pi}_t$ is a predetermined state variable, we include the auxiliary variable $\hat{x}_t := \hat{\pi}_{t+1}$ in the vector of costate variables, hence:

$$\boldsymbol{\lambda}_t := \left[\hat{\lambda}_{t,1,1}, \dots, \hat{\lambda}_{t,T-1,m}, \hat{x}_t, \hat{g}_t\right]'.$$

In the case of the Phillips curve (65) $\hat{v}_t^1 := \hat{\pi}_{t-1}$ is an auxiliary state variable. Additional auxiliary state variables $\hat{v}_t^i := \hat{\pi}_{t-i}$ are required if taxes and pensions are imperfectly indexed to inflation. The number of these variable is $n-1$ for the Phillips curves (64) and (66) and $n-2$ in the case of equation (65). Thus,

$$\mathbf{x}_t := \left[\hat{k}_{t,2,1}, \dots, \hat{k}_{t,T,m}, \hat{m}_{t,2,1}, \dots, \hat{m}_{t,T,m}, \hat{\mathbf{v}}_t'\right]',$$

$$\hat{\mathbf{v}}_t = \begin{cases} \left[\hat{\pi}_{t-1}, \dots, \hat{\pi}_{t-(n-1)}\right]' & \text{for Phillips curve (64) if } n > 1, \\ \Big[\hat{\pi}_{t-1}, \underbrace{\hat{\pi}_{t-2}, \dots, \hat{\pi}_{t-(n-1)}}_{\text{if } n>2}\Big]' & \text{for Phillips curve (65),} \\ \Big[\hat{\pi}_t, \underbrace{\hat{\pi}_{t-1}, \dots, \hat{\pi}_{t-(n-1)}}_{\text{if } n>1}\Big]' & \text{for Phillips curve (66).} \end{cases}$$

No changes are required for the vector of controls $\mathbf{u}_t$.

The system of equations (35a) consists of the log-linearized definitions of market income (see (40a)), the log-linearized definitions of disposable income, LL9

$h = 1, \dots, m,$ (70a) LL9a

$s = 1, \dots R-1:$

$$y^d_{s,h}\hat{y}^d_{t,s,h} - (1-\tau'(y_{s,h}))y_{s,h}\hat{y}_{t,s,h} - trs\,\widehat{trs}_t = -\tau'(y_{s,h})y_{s,h}\sum_{i=0}^{n-1}\hat{\pi}_{t-i},$$

$s = R, \dots, T:$

$$y^d_{s,h}\hat{y}^d_{t,s,h} - (1-\tau'(y_{s,h}))y_{s,h}\hat{y}_{t,s,h} - trs\,\widehat{trs}_t = -\left(\tau'(y_{s,h})y_{s,h} + pens_h\right)\sum_{i=0}^{n-1}\hat{\pi}_{t-i},$$

the log-linearized first-order conditions (46a) (see (56a)), the log-linearized first-order conditions (69a),

$$h = 1, \dots, m, \tag{70b}$$
$$s = 1, \dots, R-1:$$
$$\eta \frac{n_{s,h}}{1-n_{s,h}} \hat{n}_{t,s,h} - \hat{w}_t + \frac{\tau''(y_{s,h})}{1-\tau'(y_{s,h})} y_{s,h} \hat{y}_{t,s,h} = \hat{\lambda}_{t,s,h} - \frac{\tau'(y_{s,h}) + \tau''(y_{s,h}) y_{s,h}}{1-\tau'(y_{s,h})} \sum_{i=0}^{n-1} \hat{\pi}_{t-i},$$

the m log-linearized budget constraints of the T-year old households (56b), the log-linearized factor market equilibrium conditions (13) with time dependent marginal costs g_t,

$$\alpha \hat{N}_t - \alpha \hat{K}_t + \hat{w}_t = \hat{g}_t + \hat{Z}_t, \tag{70c}$$
$$(\alpha - 1)\hat{N}_t + (1-\alpha)\hat{K}_t + \hat{r}_t = \hat{g}_t + \hat{Z}_t, \tag{70d}$$

the log-linearized equation for aggregate profits (16)

$$\hat{\Omega}_t - \hat{Y}_t = (1-\epsilon)\hat{g}_t, \tag{70e}$$

equations (40h)-(40l), the log-linearized definition of aggregate money balances (56c), the log-linearized definition of aggregate taxes,

$$Tax\, \widehat{Tax}_t - \sum_{s=1}^{T} \sum_{h=1}^{m} \psi_s \nu_h \tau'(y_{s,h}) y_{s,h} \hat{y}_{t,s,h} = \sum_{s=1}^{T} \sum_{h=1}^{m} \psi_s \nu_h \tau'(y_{s,h}) y_{s,h} \sum_{i=0}^{n-1} \hat{\pi}_{t-i}, \tag{70f}$$

aggregate transfers,

$$\begin{aligned} Trs\, \widehat{Trs}_t - (1-\xi) Tax\, \widehat{Tax}_t - (1-\xi Beq)\, \widehat{Beq}_t - (1-\xi) Seign\, \hat{M}_t \\ = -(1-\xi) Seign\, \hat{\pi}_t + (1-\xi) Pens \sum_{i=0}^{n-1} \hat{\pi}_{t-i} + (1-\xi) M \hat{\theta}_t, \end{aligned} \tag{70g}$$

equations (56f), (40n), and (40o), where $G\hat{G}_t$ is now given by

$$\begin{aligned} G\hat{G}_t - \xi Tax\, \widehat{Tax}_t - \xi Beq\, \widehat{Beq}_t - \xi Seign\, \hat{M}_t = -\xi Seign\, \hat{\pi}_t + \xi Pens \sum_{i=0}^{n-1} \hat{\pi}_{t-i} \\ + \xi M \hat{\theta}_t. \end{aligned} \tag{70h}$$

The dynamic equations consist of (57a), (57b), (57d), the log-linearized Euler equations for capital (69b):

$$
\begin{aligned}
& h = 1, \ldots, m : \\
& s = 1, \ldots, T-1 : \\
& \quad \mathbb{E}_t \hat{\lambda}_{t+1,s+1,h} - \hat{\lambda}_{t,s,h} \\
& \quad - \beta \frac{\phi_{s+1}\lambda_{s+1,h}}{\lambda_s}(r-\delta)\left[\tau'(y_{s+1,h}) + \tau''(y_{s+1,h})y_{s+1,h}\right]\sum_{i=0}^{n-1}\mathbb{E}_t\hat{\pi}_{t+1-i} \\
& \quad = -\beta\frac{\phi_{s+1}\lambda_{s+1,h}}{\lambda_{s,h}}(1-\tau'(y_{s+1,h}))r\mathbb{E}_t\hat{r}_{t+1} \\
& \qquad + \beta\frac{\phi_{s+1}\lambda_{s+1,h}}{\lambda_{s,h}}(r-\delta)\tau''(y_{s+1,h})y_{s+1,h}\mathbb{E}_t\hat{y}_{t+1,s+1,h},
\end{aligned}
\tag{71a}
$$

one of the Phillips curve equations (64)-(66), and the definitions of the auxiliary variables:

$$
\begin{aligned}
& i = 1, \ldots, n-1, \\
& \hat{v}_t^i = \hat{\pi}_{t-i}.
\end{aligned}
\tag{71b}
$$

6.5 Sensitivity Analysis

Phillips Curves. Table 5 provides information on the roles played by the different Phillips curves given in equations (64)-(66). The summary statistics are from simulations that assume no further nominal rigidities than those present in the Phillips curves, i.e. $n = 0$. We use a moderate degree of price stickiness. $\phi = 0.25$ implies that prices change about every four month. This is in line with evidence provided by Bils and Klenow (2004) and Klenow and Kryvstov (2005).

If prices are set before the monetary shock, (equation (66)), even for the small value of $\phi = 0.25$, output, investment and hours are unrealistically volatile. For example, the standard deviation of output is equal to 19.81 in this case in our model, while Cooley and Hansen (1995), Table 7.1, find a value equal to 1.72 for the US economy. Inflation is less volatile than empirically observed (for the U.S. $\sigma_\pi = 0.57$, see Cooley and Hansen (1995), Table 7.1.) but displays a high degree of persistence ($r_\pi = 0.6$). For both the Phillips curves (64) and (65) inflation is too volatile with first-order autocorrelations almost equal to zero.

The main differences between the purely forward looking Phillips curve (64) and the Phillips curve (65) are found in the labor market statistics. Due to the effect of lagged inflation hours overshot in the second period after the shock and, thus, are more than twice as volatile under the Phillips curve (A_PK1 65) as compared to the simulations that use the Phillips curve (A_PK0 64). As a consequence, the distribution of market income is more than twice as volatile under the Phillips curve (A_PK1 65) than under the purely forward looking Phillips curve (A_PK0 64). Even if the government spends its extra revenues on consumption market income is still 67 percent more volatile.

Tab5

Table 5

Second Moments from Model 4 for Different Phillips Curves

Phillips Curve	PK0 (25)			PK1 (26)			PK2 (27)		
				$\xi = 0$					
Variable	s_x	r_{xy}	r_x	s_x	r_{xy}	r_x	s_x	r_{xy}	r_x
Output	0.91	1.00	0.70	0.94	1.00	0.62	19.81	1.00	−0.07
Consumption	0.39	0.98	0.72	0.39	0.95	0.72	0.83	0.42	0.70
Investment	3.82	0.99	0.68	4.15	0.99	0.53	126.77	1.00	−0.07
Effective Labor	0.21	0.12	−0.05	0.47	0.30	−0.40	31.69	1.00	−0.07
Hours	0.18	−0.07	−0.02	0.40	0.22	−0.39	26.84	1.00	−0.07
Real Wage	1.06	0.87	0.47	1.66	0.75	−0.07	96.15	1.00	−0.07
Transfers	4.79	0.38	0.37	4.91	0.44	0.32	37.10	1.00	−0.03
Inflation	1.64	0.05	−0.06	1.62	0.19	−0.05	0.06	−0.26	0.58
Marginal Costs	0.39	0.51	0.51	0.39	0.46	0.54	2.93	0.72	0.60
Gini Ratio									
Wealth	0.13	−0.05	0.91	0.13	0.03	0.86	2.68	0.27	0.70
Market Income	0.10	0.01	0.02	0.22	0.25	−0.36	9.46	0.18	−0.07
Disposable Income	0.62	−0.19	0.39	0.60	−0.23	0.44	15.44	−0.15	−0.08
				$\xi = 1$					
Output	0.95	1.00	0.68	1.02	1.00	0.55	22.70	1.00	−0.07
Consumption	0.31	0.93	0.73	0.31	0.89	0.74	0.73	0.42	0.70
Investment	4.61	0.65	0.63	4.63	0.67	0.67	116.03	1.00	−0.08
Effective Labor	0.31	0.37	0.01	0.64	0.48	−0.38	36.12	1.00	−0.07
Hours	0.20	0.31	0.00	0.41	0.45	−0.39	22.99	1.00	−0.07
Real Wage	1.06	0.90	0.46	1.67	0.81	−0.08	82.75	1.00	−0.07
Government Spending	4.80	0.45	0.37	4.94	0.52	0.31	36.59	1.00	−0.03
Inflation	1.70	0.15	−0.06	1.67	0.31	−0.04	0.06	−0.26	0.56
Marginal Costs	0.81	−0.01	0.51	0.81	−0.05	0.53	2.20	0.21	0.84
Gini Ratio									
Wealth	0.05	−0.14	0.91	0.06	−0.08	0.87	2.19	0.31	0.69
Market Income	0.03	0.77	0.33	0.05	0.72	−0.19	10.17	−0.42	−0.09
Disposable Income	0.05	0.65	0.45	0.08	0.13	−0.08	12.36	−0.48	−0.09

Notes: Second moments from HP-filtered simulated time series. The numbers are averages from 100 simulations. The length of each indivdual time series is 150 quarters. s_x:=standard deviation of variable x, r_{xy}:=cross correlation of variable x with output, r_x:=first order autocorrelation of variable x. The equation numbers refer to the Phillips curve equations. $\phi = 0.25$, $n = 0$, $m = 3$.

Degree of Price Stickiness. Table 6 considers increasing degrees of nominal rigidity φ in the case of the forward and backward looking Phillips curve (65). Again, the simulations do not embed imperfect indexation of taxes and pensions.

Table 6

Second Moments from Model 4 for Phillips Curve (26)

	$\xi = 0$								
	$\varphi = 0.25$			$\varphi = 0.50$			$\varphi = 0.75$		
Variable	s_x	r_{xy}	r_x	s_x	r_{xy}	r_x	s_x	r_{xy}	r_x
Output	0.94	1.00	0.62	1.47	1.00	0.04	4.59	1.00	−0.24
Consumption	0.39	0.95	0.72	0.39	0.69	0.71	0.43	0.55	0.60
Investment	4.15	0.99	0.53	8.19	0.98	−0.15	28.54	1.00	−0.28
Effective Labor	0.47	0.30	−0.40	1.90	0.79	−0.38	7.25	0.98	−0.29
Hours	0.40	0.22	−0.39	1.61	0.78	−0.38	6.12	0.98	−0.29
Real Wage	1.66	0.75	−0.07	5.84	0.87	−0.35	22.08	0.99	−0.28
Transfers	4.91	0.44	0.32	6.01	0.76	0.14	11.17	0.96	−0.09
Inflation	1.62	0.19	−0.05	1.51	0.66	0.00	1.12	0.82	0.16
Marginal Costs	0.39	0.46	0.54	0.37	0.23	0.67	0.54	0.57	0.62
Gini Ratio									
Wealth	0.13	0.03	0.86	0.25	0.41	0.59	0.81	0.48	0.51
Market Income	0.22	0.25	−0.36	0.84	0.78	−0.37	3.05	0.92	−0.25
Disposable Income	0.60	−0.23	0.44	0.52	−0.19	0.55	1.43	0.40	−0.06

Notes: Second moments from HP-filtered simulated time series. The numbers are averages from 100 simulations. The length of each indivdual time series is 150 quarters. s_x:=standard deviation of variable x, r_{xy}:=cross correlation of variable x with output, r_x:=first order autocorrelation of variable x. $n = 0$, $m = 2$.

If prices are becoming more sticky, the volatility of output, investment, and hours increases while the volatility of inflation decreases. For $\varphi = 0.75$, the standard deviations of these variables are unrealistically large. In order to have both a high degree of nominal rigidity and plausible degrees of volatility of major macroeconomic aggregates, one has to introduce frictions into the process of capital accumulation – as, for example, in the representative agent model of Heer and Maußner (2009c).

Degree of Heterogeneity. Table 7 considers the distributional effects if agents within a generation do not differ with respect to productivity. In our model, this is corresponds to $m = 1$.

The volatility of the Gini ratios of market and disposable income for both $\xi = 0$ and $\xi = 1$ is higher than in our benchmark model with $m = 3$ different types of

Table 7
Gini Ratios for $m = 1$ and $m = 3$

Variable	$m=1$			$m=3$		
	s_x	r_{xy}	r_x	s_x	r_{xy}	r_x
				$\xi = 0$		
Wealth	0.20	0.07	0.85	0.24	0.08	0.90
Market Income	0.48	0.06	−0.29	0.22	0.25	−0.26
Disposable Income	2.53	0.04	−0.08	1.39	0.12	0.62
				$\xi = 1$		
Wealth	0.17	−0.11	0.87	0.07	−0.14	0.89
Market Income	0.58	−0.04	−0.19	0.08	0.64	0.44
Disposable Income	2.08	−0.11	−0.30	0.21	0.21	0.45

Notes: Second moments from HP-filtered simulated time series. The numbers are averages from 100 simulations. The length of each individual time series is 150 quarters. s_x:=standard deviation of variable x, r_{xy}:=cross correlation of variable x with output, r_x:=first order autocorrelation of variable x. Phillips curve (26), $n = 4$, $\varphi = 0.25$.

productivity. This indicates that the redistributive effects between the younger and poorer households on the one hand and the older and richer households on the other hand are partly compensated by the redistribution between the poor and the rich within the same generation.

6.6 The Representative Agent Version of Model 4

Using the tax function (67) the first-order condition (60b) of the household's problem stated in (58) becomes:

$$\eta_0(1-N_t)^{-\eta} = \lambda_t w_t \left[1 - \tau'\left(\frac{\Pi_{i=0}^{n-1}\pi_{t-i}}{\pi^n}(Y_t - \delta K_t)\right)\frac{\Pi_{i=0}^{n-1}\pi_{t-i}}{\pi^n}\right], \tag{72}$$

and (60c) changes to

$$\lambda_t = \beta \mathbb{E}_t \lambda_{t+1}\left\{1 + \left[1 - \tau'\left(\frac{\Pi_{i=0}^{n-1}\pi_{t+1-i}}{\pi^n}(Y_{t+1} - \delta K_{t+1})\right)\frac{\Pi_{i=0}^{n-1}\pi_{t+1-i}}{\pi^n}\right](r_{t+1} - \delta)\right\}. \tag{73}$$

Note, the model has the same stationary equilibrium as in the representative-agent version of Model 3. The log-linear model consists of the following to sets of equations:

LL6a

$$
\begin{aligned}
[\gamma(1-\sigma)-1]\hat{C}_t &= -(1-\gamma)(1-\sigma)\hat{m}_t \\
&\quad + (1-\gamma)(1-\sigma)\hat{\pi}_t + \hat{\lambda}_t, && \text{(74a)}\\
\frac{\eta N}{1-N}\hat{N}_t + \frac{\tau'' Y}{1-\tau'}\hat{Y}_t - \hat{w}_t &= \frac{\tau'' I}{1-\tau'}\hat{K}_t + \hat{\lambda}_t \\
&\quad - \frac{\tau' + \tau''(Y-I)}{1-\tau'}\sum_{i=0}^{n-1}\hat{\pi}_{t-i}, && \text{(74b)}\\
\alpha\hat{N}_t + \hat{w}_t &= \alpha\hat{K}_t + \hat{g}_t + \hat{Z}_t, && \text{(74c)}\\
(\alpha-1)\hat{N}_t + \hat{r}_t &= (\alpha-1)\hat{K}_t + \hat{g}_t + \hat{Z}_t, && \text{(74d)}\\
(\alpha-1)\hat{N}_t + \hat{Y}_t &= \alpha\hat{K}_t + \hat{Z}_t, && \text{(74e)}\\
(C/I)\hat{C}_t - (Y/I)\hat{Y}_t + \hat{I}_t + (G/I)\hat{G}_t &= 0, && \text{(74f)}\\
\frac{-\xi\tau' Y}{G}\hat{Y}_t + \hat{G}_t &= -\frac{\xi\tau' I}{G}\hat{K}_t + \frac{\xi(\theta-1)(M/P)}{G}\hat{m}_t \\
&\quad - \frac{\xi(\theta-1)(M/P)}{G}\hat{\pi}_t + \frac{\xi\theta(M/P)}{G}\hat{\theta}_t \\
&\quad + \frac{\xi\tau'(Y-I)}{G}\sum_{i=0}^{n-1}\hat{\pi}_{t-i}. && \text{(74g)}
\end{aligned}
$$

LL6b and:

$$
\begin{aligned}
&\hat{K}_{t+1} + (\delta-1)\hat{K} &&= \delta\hat{I}_t, && \text{(75a)}\\
&\beta(r-\delta)\tau'' I\hat{K}_{t+1} + \mathbb{E}_t\hat{\lambda}_{t+1} - \hat{\lambda}_t && && \text{(75b)}\\
&\quad - \beta(r-\delta)[\tau' + \tau''(Y-I)]\sum_{i=0}^{n-1}\mathbb{E}_t\hat{\pi}_{t+1-i} &&= -\beta(1-\tau')rE_t\hat{r}_{t+1} \\
& &&\quad + \beta\tau''(r-\delta)YE_t\hat{Y}_{t+1}, \\
&\mathbb{E}_t\hat{\lambda}_{t+1} - \frac{\beta}{\theta}\mathbb{E}_t\hat{\pi}_{t+1} + \frac{\beta-\theta}{\theta}\hat{m}_{t+1} - \hat{\lambda}_t &&= \frac{\beta-\theta}{\theta}\mathbb{E}_t\hat{C}_{t+1}, && \text{(75c)}\\
&\hat{m}_{t+1} - \hat{m}_t + \hat{\pi}_t &&= \hat{\theta}_t, && \text{(75d)}
\end{aligned}
$$

to which we must add one of the Phillips curve equations (64), (65), or (66).

To match the canonical system (35) we define the vectors $\mathbf{x}_t$ and $\boldsymbol{\lambda}_t$ in the case of the Phillips curve (64) as follows

$$
\begin{aligned}
\mathbf{x}_t &:= \left[\hat{K}_t, \hat{m}_t, \hat{v}_t^1, \dots, \hat{v}_t^{n-1}\right]', \\
\boldsymbol{\lambda}_t &= \left[\hat{\lambda}_t, \hat{\pi}_t, \hat{g}_t\right]',
\end{aligned}
$$

where $\hat{v}_t^i = \hat{\pi}_{t-i}$, $i = 1, \dots, n-1$ for $n > 0$. In the case of the Phillips curve (65) we can use the same definitions. However, the auxiliary variable $\hat{v}_t^1 = \hat{\pi}_t$ now appears even for

$n = 0$. In the model with the Phillips curve (66) the definitions are

$$\mathbf{x}_t := \left[\hat{K}_t, \hat{m}_t, \pi_t, \hat{v}_t^1, \ldots, \hat{v}_t^{n-1}\right]',$$
$$\boldsymbol{\lambda}_t = \left[\hat{\lambda}_t, \hat{v}_t^n, \hat{g}_t\right]',$$

where $\hat{v}_t^i = \hat{\pi}_{t-i}$ for $n > 0$ and $i = 1, 2, \ldots, n-1$, and $\hat{v}_t^n = \pi_{t+1}$.

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