On the modeling of the income distribution business cycle dynamics

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Abstract:
Empirically, the income share is procyclical for the low-income groups and acyclical for the top 5%. To generate this kind of behavior in a DGE business cycle model, we consider overlapping generations and elastic labor supply in addition to those elements considered by Castañeda et al. (1998). However, these features do not help to constitute a major improvement vis-a-vis their model.

1 Introduction

Castañeda et al. (1998) document that the US income distribution is highly, but not perfectly procyclical for the low income quintiles, countercyclical for the top 60-95%, and acyclical for the top 5%. They also present a dynamic general equilibrium model with infinitely-lived agents and unemployment risk that is able to replicate the movements of the lower income quintiles. However, they fail to replicate the income dynamics of the very rich. In addition, the share of the lowest income quintiles is almost perfectly correlated with real output in their model. In the next section, we present a simple business cycle model with overlapping generations, elastic labor supply, and unemployment.[1] In this model, the almost perfect correlation of the lower income quintiles with output is reduced as the income-rich agents have a more elastic labor supply than the income-poor. However, the share of the top 5% of the income earners is almost perfectly negatively correlated with real output. The model is presented in Section 3, while our results and conclusion are presented in Section 3.

2 The model

Households live 70 periods. Periods are equal to one year. Households are born at age 1 (corresponding to real life-time age 20). Each generation is of measure 1/70. The first 45 periods, they are working, the last 35 periods, they are retired and receive pensions. Households maximize life-time utility at age 1 in period t:

$$\sum_{s=1}^{70} \beta^{s-1} \frac{\left(c_{t+s-1}^{\gamma} l_{t+s-1}^{1-\gamma}\right)^{1-\eta} - 1}{1-\eta},$$

where s, c, and l denote age, consumption, and leisure.

The total time endowment is equal to one and allocated between leisure l and work n, $n+l = 1$. The worker's labor productivity $e(\epsilon, z, s) = \epsilon z e^{\bar{y}_s}$ depends on the agent's permanent efficiency type $\epsilon \in \{\epsilon_1, \epsilon_2\}$, his idiosyncratic stochastic productivity $z \in \{z_1, z_2\}$, and his age s.[2] During working age, $s = 1, \ldots, 44$, the process z_s is a Markov chain. Following Krueger and Ludwig (2006), we choose $\{\epsilon_1, \epsilon_2\} = \{0.57, 1.43\}$, $\{z_1, z_2\} = \{0.727, 1.273\}$, and

$$\pi(z'|z) = Prob\{z_{s+1} = z'|z_s = z\} = \begin{pmatrix} 0.98 & 0.02 \\ 0.02 & 0.98 \end{pmatrix}. \tag{1}$$

The age-efficiency $\bar{y}_s$ profile is taken from Hansen (1993). At age $s = 1$, an equal measure of low-productivity and high-productivity workers is born. We assume that households in each generation are of measure 1/70

[1] In addition, we are able to model the observed income and wealth heterogeneity.

[2] For a more detailed description of the age-efficiency profiles please see Heer and Maussner (2009), Sections 9.3.2 and 10.2.2.

In addition, the household faces the risk of unemployment that is independent of his age, his efficiency type, his stochastic productivity, and his previous employment status (employed, unemployed). In a boom (recession), the probability to get unemployed amounts to 4% (12%). If the household is unemployed, his leisure is equal to one, $l_t = 1$, and he receives unemployment insurance payments w_{UI} that are constant over the business cycle and independent of his individual productivity $e(\epsilon, z, s)$. Unemployment insurance is financed by a contribution rate τ_{UI} on wage income. In old age, agents receive pension payments b that is also constant.

The working agent of age s faces the following budget constraint in period t for age $s = 1, \ldots, 45$:

$$k_{t+1}^{s+1} = \begin{cases} (1 + r_t - \delta)k_t^s + (1 - \tau_{w,t} - \tau_{UI,t})e(\epsilon, z, s)w_t n_t^s - c_t^s & \text{if employed,} \\ (1 + r_t - \delta)k_t^s + w_{UI} - c_t^s & \text{if unemployed.} \end{cases}$$

Capital depreciates at the rate δ.

The budget constraint of the retired worker is given by

$$k_{t+1}^{s+1} = (1 + r_t)k_t^s + b - c_t^s, \quad s = 46, \ldots, 70.$$

with $k_t^1 \equiv 0$ and $l_t^s = 1$ for $s \geq 46$.

Production Y_t is characterized by constant returns to scale and assumed to be Cobb-Douglas:

$$Y_t = A_t F(K_t, N_t) = A_t K_t^{\alpha} N_t^{1-\alpha}.$$

Firms are competitive and maximize profits $\pi_t = Y_t - r_t K_t - w_t N_t$ such that factor prices are given by:

$$w_t = (1 - \alpha)A_t K_t^{\alpha} N_t^{-\alpha}, \tag{2a}$$

$$r_t = \alpha A_t K_t^{\alpha-1} N_t^{1-\alpha}. \tag{2b}$$

In equilibrium, the following conditions hold:

1. Households maximize their intertemporal utility.
2. In a factor market equilibrium, factors are rewarded with their marginal product presented by (2).
3. The government budget is balanced in every period t.
4. Individual and aggregate behavior are consistent. In particular,

$$N_t = \sum_{s=1}^{45} \frac{1}{70} n_t^s, \tag{3a}$$

$$K_t = \sum_{s=1}^{70} \frac{1}{70} k_t^s. \tag{3b}$$

5. The goods market clears.

Calibration and computation We choose the parameter values $\beta = 0.99$, $\eta = 2.0$, $\gamma = 0.28$, $\alpha = 0.35$, $\delta = 0.08$ that are standard in the business cycle literature.[3] Following Storesletten et al. (2007), the aggregate technology level $A_t \in \{A_1, A_2\} = \{0.98, 1.02\}$ follows a 2-state Markov process:

$$\pi(A'|A) = Prob\{A_{t+1} = A'|A_t = A\} = \begin{pmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{pmatrix}. \tag{4}$$

The replacement ratio of pensions and unemployment benefits relative to net wage earnings are both set equal to 30%.

The calibration implies an average labor supply approximately equal to $\bar{n} = 0.3$ and a Gini coefficient of gross income (wealth) equal to 0.50 (0.58) in good accordance with empirical observations.

The computation is based upon the algorithm of Krusell and Smith (1998) and follows Storesletten et al. (2007). Furthermore, as we also model endogenous employment, agents have to project effective labor N'. We find that $N' = exp(a_0 + a_1 \ln(K') + a_2 \mathbf{1}_{A'=A_1} + a_3 \mathbf{1}_{A'=A_1} \ln(K'))$ is a forecasting function with an R^2 almost identical to one.[4]

Table 1: Correlation of output with income shares

	0-20%	20-40%	40-60%	60-80%	80-95%	95-100%
US	0.53	0.49	0.31	-0.29	-0.64	0.00
Castañeda et al. (1998)	0.95	0.92	0.73	-0.56	-0.90	-0.84
our model	0.12	0.85	0.95	-0.91	-0.84	-0.89

Notes: Entries in rows 1 and 2 are reproduced from Table 4 in Castañeda et al. (1998). Annual logarithmic output has been detrended using the Hodrick-Prescott filter with smoothing parameter $\lambda = 100$.

3 Results

Table 1 presents our results. In the first entry row, we display the empirical correlations of output with the 1st, 2nd, 3rd, and 4th income quintiles, and the 80-95% and 95-100% income groups for the US economy, respectively.[5] In the second row, you find the values as

[3]See, for example, Heer and Maussner (2009).

[4]The Gauss computer program is available from the author upon request.

[5]The estimates are reproduced from Table 4 in Castañeda et al. (1998). In all computations presented in Table 1, log income has been detrended using the Hodrick-Prescott filter with smoothing parameter $\lambda = 100$.

resulting from the simulation of the most preferred model of Castañeda et al. (1998). The last two lines display the values obtained from the simulation of our economy.[6] Obviously, the income share of the first is now less cyclical than in Castañeda et al. (1998) and even acyclical, while the almost perfect correlation of the top 5% income group is still almost perfectly negatively correlated with real output. We therefore conclude that our business cycle model of the income distribution with pensions and endogenous labor is not a major improvement over the model of Castañeda et al. (1998).

References

Castañeda, A., J. Díaz-Giminénez, J.-V. Ríos-Rull, 1998, Exploring the income distribution business cycle dynamics, *Journal of Monetary Economics*, vol. 42, 93-130.

Hansen, G., 1993, The cyclical and secular behavior of the labor input: comparing efficiency units and hours worked, *Journal of Applied Econometrics 8*, 71-80.

Heer, B., and A. Maussner, 2009, *Dynamic General Equilibrium models: Computational Methods and Applications*, 2nd ed., Springer-Verlag, Berlin.

Krueger, D., and A. Ludwig, 2006, On the Consequences of Demographic Change for Rates of Returns to Capital, and the Distribution of Wealth and Welfare, *Journal of Monetary Economics*, vol. 54, 49-87.

Krusell, P., and A. Smith, 1998, Income and Wealth Heterogeneity in the Macroeconomy, *Journal of Political Economy*, vol. 106, 867-96.

Storesletten, K., C.I. Telmer, and A. Yaron, 2007, Asset pricing with idiosyncratic risk and overlapping generations, *Review of Economic Dynamics*, vol. 10, 519-548.

[6] We have also experimented with a CES production function that is not Cobb-Douglas and with pensions and unemployment benefits that are proportional to the efficiency types of the workers. However, the fit did not improve.